Total No. of printed pages = 6

28/11/19

MA 171103

BINA CHOWDHURY CENTRAL LIBRARY
(GIMT & GIPS)
Azara, Hatkhowapara,
Guwahati -781017

Roll No. of candidate

2019

B.Tech. 1st Semester End-Term Examination

ENGINEERING MATHEMATICS – I

(New Regulation & New syllabus)

(w.e.f. 2017–2018)

Full Marks – 70 Time – Three hours

The figures in the margin indicate full marks for the questions.

Answer Question No. 1 and any *four* from the rest.

1. Answer the following (MCQ/Fill in blanks) : $(10 \times 1 = 10)$

(i) If $y = (x+a)^n$, then $y_n =$

(a) 0 (b) $n!$

(c) $a^n n!$ (d) $a^n x$

(ii) The function $u = \frac{x+y}{\sqrt{x}+\sqrt{y}}$ is a homogeneous

(iii) If $z = f\left(\frac{x}{y}\right)$, then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} =$

(a) 0 (b) $f\left(\frac{x}{y}\right)$

(c) $f'\left(\frac{x}{y}\right)$ (d) None of the above

(iv) If $x = r\cos\theta$, $y = r\sin\theta$ then $\frac{\partial(x, y)}{\partial(r, \theta)} =$

(a) 1 (b) r

(c) $r\cos\theta$ (d) $r\sin\theta$

(v) The value of $\int_0^{\pi/2} \sin^6 x dx$ is equal to

(a) $\frac{5\pi}{12}$ (b) $\frac{5\pi}{32}$

(c) $\frac{5}{32}$ (d) None of the above

(vi) The value of $\sqrt{\frac{3}{2}}$ is

(a) $\sqrt{\pi}$ (b) $\frac{1}{2}\sqrt{\pi}$

(c) $\frac{3}{2}\sqrt{\pi}$ (d) $\frac{3}{4}\sqrt{\pi}$

(vii) The volume of the sphere generated by the revolution of the circle $x^2 + y^2 = a^2$ about the y-axis is

(a) πa^2 (b) $2\pi a^2$

(c) $\frac{4}{3}\pi a^2$ (d) $\frac{4}{3}\pi a^3$

(viii) The integrating factor of the equation $\frac{dy}{dx} + y = e^x$ is

(a) x (b) $\frac{1}{x}$

(c) e^x (d) $\log x$

(ix) The solution of the differential equation $y = xp + \frac{a}{p^2}$ is

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(a) $\frac{a}{c^2}$ (b) xc

(c) $xc + \frac{a}{c^2}$ (d) $x + \frac{a}{c^2}$

(x) The complementary function of the equation $\frac{d^2y}{dx^2} - y = \cos x$ is ————————.

2. (a) Find y_n if $y = \sin 2x \cos 3x$. (3)

(b) Expand $(1+x)^4$ in Maclaurin's series. (5)

(c) If $y = \tan^{-1} x$, then show that (2 + 5 = 7)

(i) $(1+x^2)y_1 = 0$

(ii) $(1+x^2)y_{n+1} + 2nxy_n + n(n-1)y_{n-1} = 0$

3. (a) If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, show that

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = \frac{3}{x+y+z}.$$ (3)

(b) If $u = \sin^{-1}\left(\frac{x^2+y^2}{x+y}\right)$, show that $\sin u$ is a homogeneous function of degree 1 and

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \tan u.$$ (2 + 5 = 7)

(c) If $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$, $z = r\cos\theta$ prove that $\frac{\partial(x, y, z)}{\partial(r, \theta, \phi)} = r^2 \sin\theta$. (5)

4. (a) Obtain a reduction formula for

$$I_n = \int_0^{\pi/2} \cos^n x\,dx \text{ and hence evaluate } \int_0^{\pi/2} \cos^9 x\,dx$$

(5 + 2 = 7)

(b) Evaluate $\int_0^{\pi/4} \tan^6 x\,dx$. (3)

(c) Evaluate $\int_0^1 \int_0^1 \frac{dxdy}{\sqrt{1-x^2}\sqrt{1-y^2}}$. (5)

5. (a) Prove that $\sqrt{\frac{1}{2}} = \sqrt{\pi}$. (3)

(b) Evaluate $\int_0^1 x^2\left(1-x^2\right)^{\frac{7}{2}} dx$. (5)

(c) Find the volume of the solid generated by revolving about the x-axis, the area bounded by the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$; $x = 0$, $y = 0$. (7)

6. (a) Show that the equation $x^2 y dx - \left(x^3 + y^3\right)dy = 0$ is not exact. Hence find an integrating factor and solve it. (1 + 2 + 5 = 8)

(b) Find the complete solution of the equation $\left(D^2 - 5D + 6\right)y = e^x \cos 2x$, $\left[D \equiv \frac{d}{dx}\right]$. (3 + 4 = 7)

7. (a) Solve : $p = \log(px - y)$. (4)

(b) Find the particular integral of

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = e^{2x}$$ (4)

(c) Solve :

$$\frac{dx}{dt} + y = e^t$$

$$\frac{dy}{dt} - x = e^{-t}$$ (7)
