Total No. of printed pages = 4

MA 171103

18/12/18

Roll No. of candidate | | | | | | | | | | |

2018

B.Tech. 1st Semester End-Term Examination

ENGINEERING MATHEMATICS — I

(New Regulation & New Syllabus)

(w.e.f. 2017-2018)

Full Marks – 70 Time – Three hours

The figures in the margin indicate full marks for the questions.

Answer Question No. 1 and any *four* from the rest.

1. Answer the following (MCQ/Fill in the blanks): $(10 \times 1 = 10)$

(i) The 5^{th} derivative of $y = x^5$ is

(a) 0 (b) 4!

(c) x (d) 5!

(ii) The function $ax^2 + 2hxy + by^2$ is a homogeneous function of degree

(a) 0 (b) 1

(c) 2 (d) 3

[Turn over

(iii) If $f(x, y) = 0$, then $\frac{dy}{dx} =$

(a) $\frac{fy}{fx}$ (b) $\frac{fx}{fy}$

(c) $-\frac{fy}{fx}$ (d) $-\frac{fx}{fy}$

(iv) The integrating factor for $\frac{dy}{dx} + py = Q$ is

_____

(v) The value of $\int_0^{\pi/2} \sin^5\theta \, d\theta$ is _____

(vi) The solution of $(y - px)^2 = p^2$ is

(a) $y^2 = ax + a^2$

(b) $(y-a)^2 = a^2 x^2$

(c) $y^2 = a^2 x^2 + a^2$

(d) $y = ax \pm a$

(vii) Under what condition the equation

$M(x, y)\,dx + N(x, y)\,dy = 0$ become exact?

(viii) If $u = x f\left(\frac{y}{x}\right)$, then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} =$

(a) $f\left(\frac{y}{x}\right)$ (b) $x f\left(\frac{y}{x}\right)$

(c) 0 (d) $f'\left(\frac{y}{x}\right)$

(ix) The value of $\left|\frac{3}{2}\right.$ is

(a) $\sqrt{\pi}$ (b) $\frac{1}{2}\sqrt{\pi}$

(c) $\frac{3}{2}\sqrt{\pi}$ (d) $\frac{3}{4}\sqrt{\pi}$

(x) The volume of the solid generated by revolution about x-axis between $x = a$ and $x = b$ of the curve $y = f(x)$ is

(a) $\int_a^b y\,dx$ (b) $\int_a^b y^2\,dx$

(c) $\pi\int_a^b y^2\,dx$ (d) $\pi\int_a^b x^2\,dy$

2. (a) If $y = e^x x^2$, find y_n (3)

(b) Expand e^x in powers of x. (4)

(c) If $y = \sin^{-1} x$, prove that (3+5=8)

(i) $(1-x^2)\,y_2 - x\,y_1 = 0$

(ii) $(1-x^2)\,y_{n+2} - (2n+1)x\,y_{n+1} - n^2\,y_n = 0$.

3. (a) If $u = f(x+at) + g(x-at)$, show that

$$\frac{\partial^2 u}{\partial t^2} = a^2\frac{\partial^2 u}{\partial x^2}.$$ (5)

(b) If $u = x^2 + y^2 + z^2$, show that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 2u.$$ (3)

(c) Find the maxima and minima of the function $f(x, y) = x^3 + y^3 - 3xy$. (7)

4. (a) Obtain a reduction formula for $I_n = \int_0^{\pi/4} \tan^n x\,dx$ and hence prove that $n\,[I_{n+1} + I_{n-1}] = 1$ (5+5=10)

(b) Evaluate $\int_0^{\pi/2} \sin^6 x \cos^8 x\,dx$. (5)

5. (a) Prove that $\int_0^{\infty} e^{-gx} x^{3/2}\,dx = \frac{g}{4}\sqrt{\pi}$. (5)

(b) Evaluate $\int_0^1 \int_0^x \left(x^2 + y^2\right) dx\,dy$. (3)

(c) Find the volume of the solid generated by revolting the parabola $y^2 = 4ax$ about the x-axis bounded by the section $x = a$. (7)

6. (a) Find an integrating factor of the differential equation $\left(x^2 + y^2\right)dx - 2xy\,dy = 0$ and hence solve it. (2+5=7)

(b) Solve: $y - px = \tan^{-1} p$. (4)

(c) Find particular integral (PI) of the equation $\left(D^2 + 4\right)y = \cos 2x$. (4)

7. (a) Find the complete solution of $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^x$ (5)

(b) Solve: $x^2 \frac{d^2y}{dx^2} - 2y = x^3$. (5)

(c) Solve: $\frac{dx}{dt} + 2x - 3y = 0$, $\frac{dy}{dt} - 3x + 2y = 0$ (5)
