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**MA 131302**

Roll No. of candidate



**2019**

**B.Tech. 3rd Semester End-Term Examination**

**Computer Science**

**DISCRETE MATHEMATICS**

**(New Regulation)**

Full Marks – 70 Time – Three hours

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Answer Question No. 1 and any *four* from the rest.

1. Choose the correct answer : $(10 \times 1 = 10)$

(i) If $A = \{\phi, \{\phi\}\}$, then the power set of $A$ is

(a) $A$ (b) $\{\phi, \{\phi\}, A\}$

(c) $\{\phi, \{\phi\}, \{\{\phi\}\}, A\}$ (d) None of these

(ii) The function $f : Q \to Q$ defined by $f(x) = 3x + 5$, $x \in Q$ is

(a) not one-to-one

(b) not onto

(c) one-to-one and onto

(d) none of these

(iii) A group $(G, *)$ is abelian group if for all $a, b \in G$, the operation * satisfies

(a) $a * b = b * a$ (b) $a * b = e$

(c) $b * a = e$ (d) None of these

[Turn over

(iv) A binary relation '$\leq$' on a non-empty set $A$ is called a partial order on $A$ if the relation is

(a) Reflexive but not symmetric

(b) Reflexive but not anti-symmetric

(c) Reflexive but not transitive

(d) Reflexive, anti-symmetric and transitive

(v) The function $f(x) = e^{2x}$ generates the sequence

(a) $1, \frac{2}{\lfloor 1}, \frac{4}{\lfloor 2}, \frac{8}{\lfloor 3}, \ldots\ldots\ldots$

(b) $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots\ldots$

(c) $1, 1, \frac{1}{2}, \frac{1}{4}, \ldots\ldots$

(d) $\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \ldots\ldots$

(vi) The gcd of 723 and 45 is

(a) 2 (b) 4

(c) 3 (d) 5

(vii) If "p : He is intelligent" and "q : He is a player", then he is intelligent is

(a) $(p \wedge q) \rightarrow p)$

(b) $(\sim p \wedge q) \vee \sim q$

(c) $(p \vee (\sim q)) \rightarrow p$

(d) $p \rightarrow q$

(viii) If $f : R \rightarrow R$ and $f(x) = 3x - 7$, then $f^{-1}(14)$ is equal to

(a) 5 (b) 35

(c) 14 (d) 7

(ix) The subset of a countable set is

(a) uncountable (b) finite

(c) countable (d) none of theses

(x) The generators of the cyclic group $(G, X)$; where $G = \{1, -1, i, -i\}$ s are

(a) 1 (b) 1 and –1

(c) 1 and $i$ (d) i and $-i$

2. (a) For any sets $A$ and $B$ prove that $A = (B^C \cap A) \cup (A \cap B)$ (5)

(b) Let $Q$ be the set of rotational numbers and $f : Q \to Q$ be a function defined by $f(x) = 3x + 5, x \in Q$. Show that $f$ is onto and one-to-one. (5)

(c) Prove that $1 + 2 + 2^r + \ldots\ldots + 2^n = 2^{n+1} - 1$ by mathematical induction. (5)

3. (a) Draw the Hasse diagram of the poset $(S, \leq)$, where $S = \{2, 3, 6, 12, 24, 36\}$ and $x \leq y$ if s $x \mid y$ ($x$ divides $y$). (5)

(b) Prove that a non-empty subset $H$ of a group $G$ is a subgroup of $G$ if and only if $a, b \in H \Rightarrow ab^{-1} \in H$ for all $a, b \in H$. (5)

(c) Find the remainder when $6.7^{32} + 7.9^{45}$ is divided by 4. (5)

4. (a) Prove that a countable union of countable sets is countable. (5)

(b) Prove that the relation

$R = \{(x, y) : x - y =$ an even integer for all $x, y \in I\}$ is an equivalence relation. (5)

(c) Show that $(\sim q \wedge (p \rightarrow q)) \rightarrow \sim p$ is a tautology. (5)

5. (a) Solve the recurrence relation $a_n = 6a_{n-1} = 9a_{n-2}$ with initial conditions $a_0 = 1$ and $a_1 = 6$. (5)

(b) Obtain principal disjunctive notional form of $q \vee (p \vee \sim q)$. (5)

(c) Test the validity of the argument. If it rains, Tapas will be sick. If did not rain, therefore Tapass is not sick. (5)

6. (a) Symbolize the following using quantifiers :

(i) Everybody is not rich

(ii) Every natural number is either even or odd. (4)

(b) How many bit strings of length seven either begins with zeros or end with three ones? (4)

(c) Using generating functions solve the recurrence relation

$a_n = 3a_{n-1} + 2$ for all $n \geq 1$, $a_0 = 2$. (7)