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MA 181301 A

BINA CHOWDHURY CENTRAL LIBRARY
(GIMT & GIPS)
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Roll No. of candidate | | | | | | | | | | |

2019

B.Tech. 3rd Semester End-Term Examination

MATHEMATICS III–A

(New Regulation)

(w.e.f. 2017–18) and

New Syllabus

(w.e.f. 2018–19)

Full Marks – 70 Time – Three hours

The figures in the margin indicate full marks for the questions.

Answer Question No.1 and any four from the rest.

1. Fill in the blanks of the following questions ((i) – (x)).

(10 × 1 = 10)

(i) The partial differential equation obtained by eliminating f from $z = f(x^2 + y^2)$ is ______

(ii) The solution of $\dfrac{\partial^2 z}{\partial y^2} = \sin(xy)$ is ______

(iii) The solution of $pyz + qzx = xy$ is ______

[Turn over

(iv) Given that $P(A)=\frac{1}{3}, P(B)=\frac{1}{4}, P(A/B)=\frac{1}{6}$. The probability of $P(B/A)$ is ________

(v) The mean and variance of Binomial distribution are 8 and 4 respectively. Then $P(X=0)$ is ________

(vi) Let X be a poisson random variable such that $2P(X=0)=P(X=2)$. The standard deviation of X is ________

(vii) Let p be the transition matrix of a Markov chain. $p_{ij}^{(n)}$ of the matrix p^n gives the probability that the Markov chain starting in state Si will be in state Sj after ________ steps.

(viii) If $L\{f(t)\}=F(s)$, then $L\{t^n f(t)\}=$ ________

(ix) Laplace transform of $t^4 e^{-3t}$ is ________

(x) If $L^{-1}\{F(S)\}=f(t)$, then $L^{-1}\{F(GS)\}=$ ________

2. (a) Solve $\frac{\partial^2 z}{\partial x \partial y}=\sin x, \sin y$, given that $\frac{\partial z}{\partial y}=-2\sin y$ when $x=0$. (3)

(b) Form a partial differential equation by eliminations the arbitrary function from $\phi(x+y+z, x^2+y^2+z^2)=0$. (4)

(c) Solve the following boundary value problem by using the method of separation of variables which arises in the heat conduction in a rod

$\frac{\partial u}{\partial t}=c^2\frac{\partial^2 u}{\partial x^2}, 0<x<l$ given $u(0,t)=u(l,t)=0$ and $u(x,0)=\frac{100}{l}x$. (8)

3. (a) Solve the following partial differential equations

(i) $(x^2 - y^2 - z^2)p + 2xyq = 2xz$ (3)

(ii) $z^2(p^2z^2 + q^2) = 1$ (4)

(iii) $2xz - px^2 - 2qxy + pq = 0$ (4)

(b) Find $L\{t \sin \omega t\}$ and hence evaluate the following by using Laplace transform of derivatives: (4)

(i) $L\{\cot \cos \omega t + \sin \omega t\}$

(ii) $L\{2\cos \omega t - \omega t \sin \omega t\}$.

4. (a) Three urns contain 6 red, 4 black; 4 red, 6 black and 5 red, 5 black balls respectively. One of the urn is selected at random and a ball is drawn from it. If the ball drawn is red, find the probability that it is drawn from the second urn. (4)

(b) The probability that a bomb dropped from a plane will strike the target is $\frac{1}{5}$. If 6 bombs are dropped, find the probability that (4)

(i) exactly two will strike the target

(ii) atleast two will strike the target

(c) The probability that a person suffers a bad reaction from a certain injection is 0.001. Find the probability that out of 2000 individuals. (4)

(i) exactly 3

(ii) more than 2 individuals will suffer a lead reaction.

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(d) Each time a certain horse runs in a three horse race. He has probability $\frac{1}{2}$ of winning, $\frac{1}{4}$ of coming in second and $\frac{1}{4}$ of coming in third, independent of the outcome of my previous race. Give the transition matrix. (3)

5. (a) A sample of 100 dry battery cells tested to find the length of life produced the following results $\overline{x} = 12$ hours, $\sigma = 3$ hours.

Assuming the data to be normally distributed, what percentage of battery cells expected to have life (5)

(i) more than 15 hours

(ii) less than 6 hours

(iii) between 10 and 14 hours?

(Use the values:

$P(0 < z < 1) = 0.3413$ $P(0 < z < 2) = 0.4772$,
$P(0 < z < 0.67) = 0.2487$.

(b) (i) A random variable X has the following probability distribution

X	0	1	2	3	4	5	6	7
P(X)	a	4a	3a	7a	8a	10a	6a	9a

(1) determine the value of a

(2) Find $P(X < 3), P(X \geq 4), P(0 < x < 5)$

(3) Find the smallest value of m for which $P(X \leq m) \leq 0.6$. (5)

Or

(ii) Three fair coins are tossed. Let X denotes the number of heads on the first two coins and let Y denotes the number of tails on the last two coins.

(1) Find the joint distribution of X and Y.

(2) Find conditional distribution of X given $Y = 1$

(3) Find $\text{Cov}(X, Y)$.

(c) By the method of least square, fit the straight line to the following data considering y as the dependent variable: (5)

x	1	2	3	4	5
y	14	27	40	55	68

6. (a) Find the Laplace transform of $f(t)$, if

$$f(t) = \begin{cases} \cos t, & 0 < t < 277 \\ 0 &, \quad t > 277 \end{cases}.$$ (3)

(b) By use of Laplace transformation technique, show that $\int_0^\infty t\, e^{-3t} \sin 2t\, dt = \frac{12}{169}$. (4)

(c) Show that $L\left\{\frac{\cos \alpha t - \cos \beta t}{t}\right\} = \frac{1}{2} \ln\left(\frac{s^2 + \beta^2}{s^2 + \alpha^2}\right)$. (4)

(d) Find the inverse Laplace transform of $\frac{s^2 + s}{(s^2 + 1)(s^2 + 2s + 2)}$. (4)

7. (a) Solve the following differential equation by the method of Laplace transformation:

$$\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 6y = 5e^t \quad \text{given} \quad y(0) = 2 \quad \text{and}$$

$y'(0) = 1$. (5)

(b) A manufacturer claims that only 4% of his products supplied by him are defective. A random sample of 600 products contained 36 defectives. Test the claim of the manufacturer. (4)

(c) Solve the partial differential equation. (4)

$$p^2 - q^2 = \frac{x^2 - y^2}{z}$$

(d) Find $L\{u(t-a)\}$, where $u(t-a)$ is Heaviside's unit step function. (2)