Total No. of printed pages = 3

CE 1817 OE 11

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Roll No. of candidate | | | | | | | | | | |

2022

B.Tech. 7th Semester End-Term Examination

CE

OPTIMIZATION TECHNIQUES

New Regulation (w.e.f 2017 – 18) & New Syllabus (w.e.f 2018 – 19)

Full Marks – 70 Time – Three hours

The figures in the margin indicate full marks for the questions.

Answer question no. 1 and any four from the rest

1. Answer any seven from below : $(7 \times 2 = 14)$

(i) The stationary points of the function $f(x) = 2x^3 - 3x^2 - 12x + 4$ are

(a) The local minimum at $x = 2$ and the local maximum at $x = -1$

(b) The local minimum at $x = -1$ and the local maximum at $x = -2$

(c) The local minimum at $x = -1$ and the local maximum at $x = 2$

(d) The local minimum at $x = 1$ and the local maximum at $x = 2$

(ii) The determinant of the Hessian matrix of the function $f(x, y) = 100(y - x^2)^2 + (1 - x)^2$ at $x = y = 1$.

(a) 100 (b) 200

(c) 300 (d) 400

(iii) The Hessian matrix of the function $f(x) = 2x_1^2 - 2x_1x_2 + x_2^2 + 2x_1 - 2x_2$ is

(a) $\begin{bmatrix} 4 & -2 \\ -2 & 2 \end{bmatrix}$ (b) $\begin{bmatrix} -4 & -2 \\ -2 & 2 \end{bmatrix}$

(c) $\begin{bmatrix} 4 & -2 \\ -2 & -2 \end{bmatrix}$ (d) $\begin{bmatrix} -4 & -2 \\ -2 & -2 \end{bmatrix}$

[Turn over

(iv) What is nonlinear optimization problem?

(v) What is canonical form of LPP?

(vi) Define Slack and Surplus variables.

(vii) What are the nonlinear optimization techniques?

(viii) Write two limitations of classical methods of optimization.

(ix) Write two bio inspired optimization methods.

2. A boundary wall has to be constructed to cover an area of 100 m^2. The outer line of the boundary wall is marked as ABCDEA as shown in the Fig. 1. The ABCDEA consists of a rectangle BCDE joined to an equilateral traingle BFA and a sector FEA of a circle with radius x meters and centre F. (2+3+3+6=14)

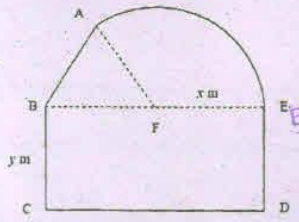

Fig. 1 Plan View of ABCDEA.

The points B, F and E lie on a straight line with FE = x meters and $10 \le x \le 25$. Given that BC = y metres, where $y > 0$.

(a) Find the area (in m^2) of the sector FEA in terms of x.

(b) Show that $y = \frac{500}{x} - \frac{x}{24}(4\pi + 3\sqrt{3})$

(c) Show that the perimeter P (in m) of the enclosure ABCDEA is given by $P = \frac{1000}{x} + \frac{x}{12}(4\pi + 36 - 3\sqrt{3})$

(d) Use calculus to find the minimum value of P and justify that the value of P obtained is a minimum.

3. The horizontal curve C of a road has the equation $y = 6 - 3x - \frac{4}{x^3}$, $x \neq 0$.

(a) Use calculus to show that the curve has a turning point M when $x = \sqrt{2}$.

(b) Find the x and y co-ordinates of the other turning point N on the curve.

(c) Find the nature of the turning points M and N. (3 + 4 + 7 = 14)

4. A manufacturer is engaged in producing two products X and Y, the profit margin of which being Rs. 15 and Rs. 45 per unit respectively. A unit of product X required 1 unit of facility A and 0.5 unit of facility B. A unit of product Y requires 1.6 unit of facility A and 2 unit of facility B and 1 unit of raw material C. The availability of total facility A, B and raw material C during a month are 240, 162 and 50 units respectively. Formulate the problem as LPP to find out the product mix that will maximize the profit of the manufacturer. Also solve the problem graphically. (6+8=14)

5. Formulate the dual of the following primal and solve the dual by dual simples method. (14)

Maximize $Z = 9x_1 + x_2$

Subject to $2x_1 + x_2 \leq 8$

$4x_1 + 3x_3 \leq 14$

$x_1, x_2 \geq 0$

6. Using simplex method, find out the solution of the LPP given below (14)

Maximize $Z = 4x_1 + 3x_2$

Subject to $x_1 + x_2 \leq 5$

$3x_1 + 2x_2 \leq 12$

$x_1, x_2 \geq 0$

7. A total of 8 available resources are to be allocated among three different investment programmes A, B and C. The return function for each programme is tabulated below: (14)

Resources	0	1	2	3	4	5	6	7	8
A	0	5	15	40	80	90	95	98	100
B	0	5	15	40	60	70	73	74	75
C	0	4	26	40	45	50	51	52	53

Using Dynamic programming technique, determine the amount of resources to be allocated to different programmes for obtaining maximum benefit from the available resources.