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MA 131101

Roll No. of candidate

2018

B.Tech. 1st Semester End-Term Examination, 2018

MATHEMATICS – I

(Old Regulation)

Full Marks – 100 Time – Three hours

The figures in the margin indicate full marks for the questions.

Answer question No. 1 and any *Six* from the rest.

1. Answer the following : (MCQ) (10 × 1 = 10)

   (i) The nth derivative of e^{mx} is ————.

   (a) 0

   (b) me^{mx}

   (c) $m^n e^{mx}$

   (d) None of the above

[Turn over

(ii) If $f(x, y) = e^{xy}$ then $f_{xy} =$ ________.

(a) $(1 + xy)\, e^{xy}$

(b) $(1 - xy)\, e^{xy}$

(c) ye^{xy}

(d) xe^{xy}

(iii) If $u \dfrac{x^2 + y^2}{x^2 - y^2}$ then $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} =$ ________.

(a) 0

(b) 1

(c) 2

(d) None of the above

(iv) $\int_0^{\pi/2} \sin^7 x \, dx =$ ________.

(a) $\dfrac{8}{15}$

(b) $\dfrac{16}{35}$

(c) $\dfrac{16}{35}\pi$

(d) $\dfrac{8}{35}\pi$

(v) The area bounded by the curve $r = f(\theta)$ and the radii vectors $\theta = \alpha$ and $\theta = \beta$ is

(a) $\int_{\alpha}^{\beta} r \, d\theta$

(b) $\int_{\alpha}^{\beta} r^2 \, d\theta$

(c) $\frac{1}{2}\int_{\alpha}^{\beta} r^2 \, d\theta$

(d) $\frac{1}{2}\int_{\alpha}^{\beta} r \, d\theta$

(vi) The volume of the solid generated by the revolution about x — axis of the area bounded by the curves $y_1 = f(x)$ and $y_2 = g(x)$ the ordinates $x = a$ and $x = b$ is

(a) $\int_{a}^{b} (y_1 - y_2) \, dx$

(b) $\int_{a}^{b} ({y_1}^2 - {y_2}^2) \, dx$

(c) $\int_{a}^{b} \pi ({y_1}^2 - {y_2}^2) \, dx$

(d) $\frac{1}{2}\int_{a}^{b} \pi ({y_1}^2 - {y_2}^2) \, dx$

(vii) The degree of the differential equation $\left(\frac{dy}{dx}\right)^3 = \sqrt{\left(\frac{d^2y}{dx^2}+1\right)}$ is

(a) 1

(b) 2

(c) 4

(d) None of the above

(viii) If $y = \log(x+1)$, then the value of $y_2(0)$ is

(a) 0

(b) 1

(c) -1

(d) None of the above

(ix) The differential equation $Mdx + Ndy = 0$ is said to be exact if

(a) $\frac{\partial M}{\partial x} = \frac{\partial N}{\partial y}$

(b) $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

(c) $\frac{\partial M}{\partial y} = -\frac{\partial N}{\partial x}$

(d) $\frac{\partial^2 M}{\partial x^2} = \frac{\partial^2 N}{\partial y^2}$

(x) An integrating factor for the differential equation $xdy - ydx = 0$ is

(a) y

(b) $\frac{1}{x}$

(c) $\frac{1}{x^2}$

(d) xy

2. (a) Find y_n if $y = \cos 2x$. (2)

(b) Expand e^x in powers of x. (3)

(c) If $y = \tan^{-1} x$ prove that $(1+x^2)\, y_{n+1} + 2nxy_n + n(n-1)\, y_{n-1} = 0$. (5)

(d) If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, prove that (5)

$$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = \frac{9}{(x+y+z)^2}.$$

3. (a) Find the value of $\int_0^{\pi/2} \sin^6 x \cos^8 x dx$. (5)

(b) If $I_{n} = \int_0^{\pi/4} \tan^n x\, dx$, prove that $I_n + I_{n-2} = \frac{1}{n-1}$. (5)

(c) Find the volume of the solid formed by the revolution of the cardiod $r = a(1 + \cos\theta)$ about the initial line. (5)

4. (a) Evaluate $\iiint (x+y+z+1)^4 \, dx\, dy\, dz$ over the tetrahedron bounded by $x=0, y=0, z=0$ and $x+y+z=1$. (5)

(b) Show that $\int_0^1 \frac{x^2}{\sqrt{1-x^4}} dx = \frac{1}{4}\beta\left(\frac{3}{4}, \frac{1}{2}\right)$. (5)

(c) By applying differentiation under integral sign, evaluate $\int_0^1 \frac{x^\alpha - 1}{\log x} dx$. (5)

5. (a) Solve $(x^2 - y^2)\, dx + 2xy\, dy = 0$. (5)

(b) Find the complete solution of $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = e^x \cos 2x$. (5)

(c) Find the integrating factor of the following differential equation and solve it

$(x^2 y) dx - (x^3 + y^3)\, dy = 0$. (5)

6. (a) The period T of a simple pendulum is $T = 2\pi\sqrt{\frac{l}{g}}$. Find the maximum error in T due to possible errors upto 1% in l and 2.5% in g. (5)

(b) Write down the conditions for a function to be maximum or minimum at a point. Show that the function $f(x, y) = x^3 + y^3 - 63(x+y) + 12xy$ is minimum at (–7, 7) and minimum at (3, 3). (10)

7. (a) Evaluate $\iint_R xy\,dx\,dy$ where R is the quadrant of the circle $x^2 + y^2 = a^2$, where $x, y \geq 0$. (5)

(b) Sketch the polar curve $x^{2/3} + y^{2/3} = a^{2/3}$. Hence find the surface area of the solid generated by revolving the curve about the initial line. (10)

8. (a) Solve the following differential equation by method of variation of parameters

$$(D^2 + 4)\,y = \tan 2x\,.$$ (6)

(b) Solve $(D^2 + D^3 - 3D^2 - 5D - 2)\,y = 3xe^{-x}$. (9)

9. (a) If $x = r\sin\theta\cos\phi, y = r\sin\theta\sin\phi$ and $z = r\cos\theta$ then prove that $\dfrac{\partial(x,y,z)}{\partial(r,\theta,\phi)} = r^2\sin\theta$. (6)

(b) If $u = x\phi(y/x) + \Psi(y/x)$, then show that (9)

(i) $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = x\phi\left(\dfrac{y}{x}\right)$.

(ii) $x^2\dfrac{\partial^2 u}{\partial x^2} + 2xy\dfrac{\partial^2 u}{\partial x\,\partial y} + y^2\dfrac{\partial^2 u}{\partial y^2} = 0$.
