Total No. of printed pages = 4

MA 181102

Roll No. of candidate

15/3 | 2022

B.Tech. 1st Semester End-Term Examination

MATHEMATICS – I

(New Regulation (w.e.f 2017–18) & New Syllabus (Group – B) (w.e.f 2018-19)

Full Marks – 70 Time – Three hours

The figures in the margin indicate full marks for the questions.

Answer question No. 1 and any *four* from the rest.

1. Answer the following questions : (10 × 1 = 10)

(i) If $y = e^{-2x}$, then y_n is

(a) $(-1)^n 2^n y$ (b) $2^n y$

(c) $-2^n y$ (d) none of these

(ii) The value of $\lim_{x \to 0} \dfrac{e^{2x} - 1}{\log(1+x)}$ is

(a) 0 (b) 1

(c) 2 (d) none of these

(iii) If $u = f\left(\dfrac{x}{y}\right)$, then $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} =$

(a) 0 (b) 1

(c) $f\left(\dfrac{x}{y}\right)$ (d) $f'\left(\dfrac{x}{y}\right)$

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(iv) The value of $\int_0^{\frac{\pi}{2}} \cos^9 x \, dx$ is

(a) $\frac{8}{15}$ (b) $\frac{32}{35}$

(c) $\frac{1}{10}$ (d) $\frac{128}{315}$

(v) The value of $\Gamma\frac{7}{2}$ (gamma function) is

(a) $\frac{7}{2}$ (b) $\frac{15}{16}$

(c) $\frac{15\pi}{16}$ (d) $\frac{5}{8}$

(vi) The volume of the solid generated by the revolution about x-axis of the area bounded by the curves $y_1 = f(x)$ and $y_2 = g(x)$ and the ordinates $x = a$ and $x = b$ is

(a) $\int_a^b (y_1 - y_2) \, dx$ (b) $\int_a^b (y_1^2 - y_2^2) \, dx$

(c) $\int_a^b \pi(y_1^2 - y_2^2) \, dx$ (d) $\frac{1}{2}\int_a^b \pi(y_1^2 - y_2^2) \, dx$

(vii) If $A = \begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix}$ satisfy Caley-Hamilton theorem, then

(a) $A^2 + 7A + I = 0$ (b) $A^2 - 7A - I = 0$

(c) $A^2 - 7A + I = 0$ (d) none of these

(viii) The equations $2x + y = 0$ and $4x + 2y = 0$ has

(a) No solution (b) Trivial solution

(c) Non-trivial solutions (d) None of these

(ix) From the following which is skew-symmetric matrix.

(a) $\begin{bmatrix} 0 & -h & -g \\ h & 0 & -f \\ g & f & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & -h & g \\ h & 0 & -f \\ g & f & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & -h & -g \\ h & 0 & -f \\ g & f & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & -h & -g \\ h & 1 & -f \\ g & f & 0 \end{bmatrix}$

(x) If Σu_n be a convergent series of positive terms, then it necessarily follows that $\lim_{n\to\infty} u_n =$ ____________

(a) ∞ (b) 0

(c) 1 (d) $\frac{1}{n}$

2. Answer the following : $(3 \times 5 = 15)$

(a) Evaluate $\int_0^\infty \frac{dx}{(1+x^2)^4}$.

(b) If $I_n = \int_0^{\frac{\pi}{4}} \tan^n x \, dx$, prove that $I_n + I_{n-2} = \frac{1}{n-1}$.

(c) Find the volume of the solid generated by revolution of the cardioid $r = a(1+\cos\theta)$ about the initial line.

3. Answer the following : $(3 \times 5 = 15)$

(a) If $y = \frac{\sin^{-1} x}{\sqrt{1-x^2}}$, show that $(1-x^2)\, y_{n+2} - (2n+3)\, xy_{n+1} - (n+1)^2 y_n = 0$.

(b) Using Maclaurin's theorem, expand $\sin x$ in an infinite series.

(c) Find the radius of curvature of a polar curve given by $r = a(1+\cos\theta)$.

4. Answer the following : $(8+4+3 = 15)$

(a) Given that $f(x) = x^2 + x$ for $-\pi < x < \pi$ find the Fourier series expansion of $f(x)$. Deduce that $\frac{\pi}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \ldots$

(b) Expand $f(x) = x$ as a half-range cosine series in $0 < x < 2$.

(c) Test the convergence of the series $\sum u_n$ where $u_n = \sqrt{n^2+1} - n$.

5. Answer the following : $(3 \times 5 = 15)$

(a) If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = \frac{3}{x+y+z}$.

(b) Apply Lagrange's method of multipliers to find the minimum value of $x^2 + y^2 + z^2$ under the condition $x + y + z = 12$.

(c) Find the area lying between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$.

6. Answer the following : (3 × 5 =15)

(a) Reduce the matrix $A = \begin{bmatrix} 3 & 1 & 4 & 6 \\ 2 & 1 & 2 & 4 \\ 4 & 2 & 5 & 8 \\ 1 & 1 & 2 & 2 \end{bmatrix}$ to echelon form and hence find its rank.

(b) Show that the vectors $(1, 1, -1)$, $(2, -3, 5)$ and $(-2, 1, 4)$ of R^3 are linearly independent.

(c) Find the eigenvalues and corresponding eigenvectors of $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$.

7. Answer the following : (3 × 5 =15)

(a) Evaluate $\int_0^1 \int_x^{\sqrt{x}} \left(x^2 + y^2\right) dy\, dx$.

(b) Find the values of a and b such that $\lim_{x \to 0} \frac{x(1 - a\cos x) + b\sin x}{x^3} = \frac{1}{3}$.

(c) Using Gauss-Jordan method find the inverse of $A = \begin{bmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ 4 & 1 & 5 \end{bmatrix}$.