Total No. of printed pages = 6

28/11/19

MA 171301

Roll No. of candidate | | | | | | | | | |

2019

B.Tech. 3rd Semester End-Term Examination

MATHEMATICS – III

(New Regulation)

(w.e.f. 2017-18)

Full Marks – 70 Time – Three hours

The figures in the margin indicate full marks for the questions.

Answer Question No. 1 and any *four* from the rest.

1. In each of the following questions, four answers are provided of which only one is correct. Choose the correct answer. (10 × 1 = 10)

   (i) The degree of the partial differential equation

   $$\left(\frac{\partial z}{\partial x}\right)^4 + \left(\frac{\partial^3 z}{dy^3}\right) = 2x\frac{\partial z}{\partial x} \text{ is}$$

   (a) 0 (b) 1

   (c) 2 (d) 3

[Turn over

(ii) The particular integral of the partial differential equation $\left(D^3 - 3D^2D' + 4D'^2\right)z = e^{x+2y}$ is

(a) $\frac{1}{27}e^{x+2y}$ (b) $27e^{x+2y}$

(c) $\frac{1}{7}e^{x+2y}$ (d) $7e^{x+2y}$

(iii) Let A and B be two events such that $P(A) = \frac{2}{3}, P(A \cap B) = \frac{17}{30}$. Then $P(A|B)$ is equal to

(a) $\frac{17}{20}$ (b) $\frac{11}{20}$

(c) $\frac{17}{30}$ (d) None of the above

(iv) Cauchy-Riemann equations are

(a) $u_x = -v_y$ and $u_y = -v_x$

(b) $u_x = v_y$ and $u_y = -v_x$

(c) $u_x = v_y$ and $u_y = v_x$

(d) $u_x = -v_y$ and $u_y = -vV_x$

(v) A function is said to be harmonic if and only if

(a) $u_{xx} + u_{yy} = 0$

(b) $u_{xy} + u_{xy} = 0$

(c) $u_{yx} + u_{yx} = 0$

(d) $u_{xyy} + u_{yy} = 0$

(vi) $\tan x$ is a periodic function of

(a) 2π (b) 3π

(c) π (d) None of the above

(vii) Which of the following is an odd function?

(a) $x^3 + 5x$ (b) $x^4 + 5x$

(c) $x^3 + 5x^2$ (d) None of the above

(viii) The mean of the binomial distribution is

(a) np (b) npq

(c) npq^2 (d) np^2q

(ix) If C is the left hand of the unit circle $|z| = 1$ from $z = -i$ to $z = i$, then $\oint_c \frac{dz}{z-a}$ is equal to

(a) $2\pi i$ (b) $4\pi i$

(c) $\frac{\pi i}{2}$ (d) None of the above

(x) Fourier co-efficient in the Fourier series for the function $f(x) = x\cos x$ in $(-\pi, \pi)$ is

(a) 1 (b) 2

(c) 0 (d) None of the above

2. (a) Form a partial differential equation by eliminating the arbitrary functions f and g from $z = xf(y) + yg(x)$. (3)

(b) Solve the partial differential equation by Lagrange's method.

$yzp - xzq = xy$

where p and q have their usual meanings. (6)

(c) Solve the partial differential equation by the method of separation of variable (6)

$$\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$$

where $u(x, 0) = 6^{-3x}$.

3. (a) Solve the following partial differential equation : (6)

$$2\frac{\partial^2 z}{\partial x^3} - 3\frac{\partial^2 z}{\partial x^2 \partial y} + 4\frac{\partial^3 z}{\partial y^3} = e^{x+2y}$$

(b) Solve the heat equation : (9)

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$$

(c is the constant of diffusivity of material) which satisfies the condition $u(0, t) = u(l, t) = 0$ and $u(x, 0) = f(x)$ at $t = 0$.

4. (a) Show that the function $e^x(\cos y + i \sin y)$ is analytic and find its derivative. (5)

(b) Show that the transformation $w = z^2$ maps the circle $|z-1| = 1$ into the cardiod $\rho = 2(1 + \cos\phi)$. (5)

(c) Determine the analytic function whose imaginary part is given by (5)

$$v = \log(x^2 + y^2) + x - 2y.$$

5. (a) A bag A contains 2 white and 4 black balls. Another bag B contains 5 white and 7 black balls. A ball is transferred from bag A to bag B. Then a ball is drawn from bag B. Find the probability that the drawn ball is white. (5)

(b) An unbiased coin is tossed six times. What is the probability that the tosses will result in (5)

(i) At most three balls

(ii) At least five tails

(iii) Exactly two heads.

(c) A manufacturer claimed that at least 95% of the equipment which he supplied to a factory conformed to specifications. An examination of a sample of 200 pieces of equipments revealed that 18 were faulty. Test his claim at significance level of 5% [$Z_{tab}(5\%) = 1.645$). (5)

6. (a) Using Cauchy integral formula, evaluate (5)

$$\int \frac{e^{-2z}}{(z+1)^3} dz$$

where C is the circle $|z| = 2$.

(b) Show by Residue theorem that (5)

$$\int_0^{2\pi} \frac{d\theta}{5 + 3\sin\theta} = \frac{\pi}{2}$$

(c) Define Poisson distribution. Show that Poisson distribution is the limiting case of Binomial distribution. (5)

7. (a) Find the Fourier series for the function $f(x) = x - x^2, -\pi < x < \pi$. Also deduce

$$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + ... = \frac{\pi^2}{12}$$ (6)

(b) Find the half range Fourier cosine series for the function : (9)

$$f(x) = \begin{cases} 2 - x & 0 \le x \le 4 \\ x - 6 & 4 < x \le 6 \\ 0 & 6 < x \le 8 \end{cases}$$
