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MA 181301B

Roll No. of candidate

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BINA CHOWDHURY CENTRE  
(GIMT & GIPS)  
Azara, Hatkhowapara,  
Guwahati - 781017

2019

B.Tech. 3rd Semester End-Term Examination

MATHEMATICS - III B

(New Regulation)

(w.e.f. 2017-2018) & (New Syllabus) (w.e.f. 2018-2019)

Full Marks - 70

Time - Three hours

The figures in the margin indicate full marks  
for the questions.

Answer question No. 1 and any *four* from the rest.

1. Answer the following : (10 × 1 = 10)

(A) Choose the correct answer :

(i) The mean of  $n$  terms is  $\bar{x}$ . If the first term is increased by 1, second by 2 and so on, then the new mean is

(a)  $\bar{x} + n$

(b)  $\bar{x} + \frac{n}{2}$

(c)  $\bar{x} + \frac{n+1}{2}$

(d) None of these

[Turn over

- (ii) In two throws of a dice probability that 6 appears at least once is
- (a)  $\frac{1}{6}$
  - (b)  $\frac{1}{4}$
  - (c)  $\frac{10}{36}$
  - (d)  $\frac{11}{36}$
- (iii) The mean of a binomial distribution is 10 and its standard deviation is 2. The value of  $p$  is
- (a) 0.4
  - (b) 0.6
  - (c) 1
  - (d) 0.8
- (iv) If  $A$  and  $B$  are any two events, then  $P$  (neither  $A$  nor  $B$ ) equals
- (a)  $1 - P(A \cup B)$
  - (b)  $P(\bar{A}) + P(\bar{B})$
  - (c)  $1 - P(A \cap B)$
  - (d)  $1 - P(A) - P(B)$
- (v) If the correlation co-efficient between  $x$  and  $y$  is 0.28, covariance between  $x$  and  $y$  is 7.6 and the variance of  $x$  is 9, then the standard deviation of  $y$  is approximately
- (a) 9.8
  - (b) 10.1
  - (c) 9.05
  - (d) 10.05

- (vi) If the angle between the two lines of regression is  $90^\circ$ , then it represents
- perfect correlation
  - perfect negative correlation
  - no linear correlation
  - none of these
- (vii) If  $P(x = 2) = 9P(x = 4) + 90P(x = 6)$ , then the standard deviation of the Poisson variate  $x$  is
- 1
  - $\sqrt{2}$
  - 2
  - 4
- (viii) Which vectors is a probability vector?

(a)  $\left(\frac{1}{2}, \frac{1}{3}, 0, \frac{1}{5}\right)$

(b)  $(3, 4, 1, -7)$

(c)  $\left(\frac{1}{4}, \frac{1}{3}, \frac{3}{4}, -\frac{1}{3}\right)$

(d)  $\left(\frac{1}{4}, \frac{1}{4}, 0, \frac{1}{2}\right)$

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(B) Fill in the blanks :

- (ix) If  $d_i$  denotes the rank difference of the  $i^{\text{th}}$  individual, then the rank correlation co-efficient between two characteristics of a group of  $n$  individuals is given by \_\_\_\_\_.
- (x) The value that separates the acceptance region from the rejection region is called the \_\_\_\_\_.

2. (a) What is the probability that a leap year selected at random will contain either 53 Thursdays or 53 Fridays or both?
- (b) The probabilities that three persons A, B, C becoming managers are  $\frac{4}{9}$ ,  $\frac{2}{9}$  and  $\frac{1}{3}$  respectively. The probabilities that the bonus scheme will be introduced if they become managers are  $\frac{3}{10}$ ,  $\frac{1}{2}$  and  $\frac{4}{5}$  respectively.
- (i) What is the probability that the bonus scheme will be introduced?
- (ii) If the bonus scheme has been introduced what is the probability that it was due to Mr. A being appointed as Manager? (2+3=5)

- (c) Find the arithmetic mean, median and mode for the following distribution

|                  |      |       |       |       |       |
|------------------|------|-------|-------|-------|-------|
| Class interval : | 0-10 | 10-20 | 20-30 | 30-40 | 40-50 |
| Frequency :      | 5    | 8     | 15    | 16    | 6     |

Which one of the three averages do you consider to be more representative and why?

3. (a) Fit a least square straight line to the following data :

|      |    |    |    |    |    |
|------|----|----|----|----|----|
| $x:$ | 2  | 7  | 9  | 1  | 5  |
| $y:$ | 13 | 21 | 23 | 14 | 15 |

Hence predict  $y(4)$ . (4)

- (b) Two dice are tossed simultaneously. Write the probability distribution for the difference of the numbers shown on the two faces. Also find its mean and variance. (4)

(c) A variable  $x$  has binomial distribution with  $n = 100$  and  $p = 0.01$ . Using suitable distribution find : (3)

(i)  $P(x = 0)$

(ii)  $P(x > 2)$  and

(iii)  $E(x)$ .

(d) Find the unit fixed probability vector of the matrix (4)

$$A = \begin{pmatrix} 0 & 1 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \end{pmatrix}.$$

4. (a) The appearing of 1 or 2 on a die is counted as a success. Four dice are thrown 243 times. Find the expected number of times of getting

(i) at least two successes

(ii) exactly three successes. (4)

(b) Calculate the correlation co-efficient and hence obtain the equations of the two regression lines from the following data : (5)

$$x: \quad 1 \quad 2 \quad 3 \quad 4 \quad 5$$

$$y: \quad 2 \quad 5 \quad 3 \quad 8 \quad 7$$

(c) A fair coin is tossed twice. Let  $x$  denotes 0 or 1 according as a head or a tail appears on the first toss. Let  $y$  denotes the number of heads which occur.

(i) Find the marginal distributions of  $x$  and  $y$

(ii) Write the joint distribution of  $x$  and  $y$

(iii) Find  $E(x)$  and  $E(xy)$ . (2+2+2=6)

5. (a) A continuous random variable  $x$  has the following probability density function :

$$f(x) = \begin{cases} k(1-x^2) & \text{for } 0 < x < 1 \\ 0 & \text{elsewhere} \end{cases}$$

Find

- (i)  $k$   
(ii)  $P(x > 0.5)$ . (1+2=3)

- (b) The regression lines between two variables  $x$  and  $y$  are given as  $2x + 3y - 6 = 0$  and  $5x + 7y - 12 = 0$ . Obtain

- (i) the mean of  $x$  and the mean of  $y$   
(ii) correlation co-efficient between  $x$  and  $y$   
(iii)  $\sigma_x$  if  $\sigma_y$  is given to be 3. (2+2+1=5)

- (c) In a sample analysis of 600 students it was found that 280 students have failed, 170 secured a 3<sup>rd</sup> class, 90 secured a second class and the rest a first class.

Do these figures support the general belief that above categories are in the ratio 4:3:2:1? (4)

- (d) Test whether the given matrix is regular stochastic or not (3)

$$B = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{2} \end{pmatrix}$$

6. (a) The mean height of 500 students in a certain college is 151 cm and the standard deviation is 15 cm. Assuming the heights to be normally distributed find how many students have heights

(i) between 120 cm and 145 cm

(ii) below 165 cm. (3+2=5)

(b) Write the conditions under which  $t$ -test is applicable. Also write one application of it. (2)

(c) It is known from the past experience that the mean weight of the products from a filling machine is 10 gms. A random sample of 10 shows that weight (in grams) of the sample products are as follows :

9.8, 10.3, 10.4, 9.6, 9.5, 9.7, 9.8, 10.4, 10.2, 10.3

Test the significance of the difference of the sample mean from the population mean. (5)

(d) The screws produced in machine A has mean length 5.2 cm with s.d. 0.4 cm while that produced by machine B has mean length 6.4 cm with s.d. 0.6 cm. Which of these is more variable? (3)

7. (a) The following values were obtained from a frequency distribution :

$$\sum f = 100, \sum fx = -100, \sum fx^2 = 5500,$$

$$\sum fx^3 = -6250 \text{ and } \sum fx^4 = 775000.$$

Find the moments about mean and hence find the moment co-efficients of skewness and kurtosis. Also discuss the nature of skewness and kurtosis. (3+2+2=7)

(b) Define :

(i) Markov process

(ii) Markov chain and

(iii) Stochastic matrix. (3)

(c) Draw the transition diagram for the following transition matrix : (3)

$$A = \begin{matrix} & \begin{matrix} a_1 & a_2 & a_3 \end{matrix} \\ \begin{matrix} a_1 \\ a_2 \\ a_3 \end{matrix} & \begin{pmatrix} 0 & 1 & 0 \\ \frac{1}{3} & 0 & \frac{2}{3} \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \end{pmatrix} \end{matrix}$$

(d) Show that correlation coefficient is the geometric mean of the two regression co-efficients. (2)

