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MA 131302 NR

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2021

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(MIMT & TIPS)
Azimkhat, Wapara,
Gowahati-781017

B.Tech. 3rd Semester End-Term Examination

CSE

DISCRETE MATHEMATICS

(New Regulation)

Full Marks – 70

Time – Three hours

The figures in the margin indicate full marks for the questions.

Answer question No. 1 and any *four* from the rest.

1. Chose the correct answer : (10 × 1 = 10)
- (i) Let $R = \{(0, 0), (0, 2), (2, 0), (2, 2), (2, 3), (3, 2), (3, 3)\}$ be the relation defined on $X = \{0, 1, 2, 3\}$. Then R is
- (a) Reflexive but not symmetric
 - (b) Symmetric but not reflexive
 - (c) Transitive but not symmetric
 - (d) Reflexive and symmetric
- (ii) A onto function is also known as
- (a) Injective function
 - (b) Surjective function
 - (c) Bijective function
 - (d) None of the above
- (iii) The order of the element i of the cyclic group $G = \{1, -1, i, -i\}$ is
- (a) 1
 - (b) 2
 - (c) 3
 - (d) 4
- (iv) $p \rightarrow q$ is equivalent to
- (a) $(\sim p \vee q)$
 - (b) $(p \vee \sim q)$
 - (c) $(\sim p \vee \sim q)$
 - (d) None of the above

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(v) The generating function of the sequence $1, 1, 1, \dots$ is

(a) $\frac{1}{1+x}$

(b) $\frac{1}{1-x}$

(c) $\frac{1}{x-1}$

(d) None of the above

(vi) Which of the following pairs are comparable in the poset $(\mathbb{Z}^+, |)$

(a) 2, 4

(b) 4, 6

(c) 3, 5

(d) 4, 5

(vii) If S and T are two subgroups of a group G , then which of the following is a subgroup?

(a) $S \cup T$

(b) $S \cap T$

(c) $S - T$

(d) $G - S$

(viii) The number of nonzero zero divisor of the ring \mathbb{Z}_3 is

(a) 0

(b) 1

(c) 2

(d) 3

(ix) The order of the symmetric group S_n is

(a) $\frac{n!}{2}$

(b) n

(c) $\frac{n}{2}$

(d) $n!$

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(x) The order of the permutation $\begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$ is

(a) 2

(b) 3

(c) 1

(d) None of the above

2. Answer the following

(a) If A , B and C are any three sets, prove that $A \times (B \cap C) = (A \times B) \cap (A \times C)$. (5)

(b) Using mathematical induction prove that

$$1^2 + 3^2 + 5^2 + \dots + (2n-1)^2 = \frac{1}{3}n(2n-1)(2n+1). \quad (5)$$

(c) If R is a relation on the set of positive integers such that " $a R b$ if and only if $a^2 + b$ is even", then prove that R is an equivalence relation. (5)

3. Answer the following:

(a) If $A = \{x \in \mathbb{R} \mid x \neq \frac{1}{2}\}$ and $f: A \rightarrow \mathbb{R}$ is defined by $f(x) = \frac{4x}{2x-1}$, then show that f is invertible. (5)

(b) The set of all positive divisors of 42 is a poset with respect to the partial order m/n (m divides n). Draw the Hasse diagram. (5)

(c) If $P(S)$ is the power set of a set S and \cup and \cap are taken as the join and meet, show that $(P(S), \subseteq)$ is a lattice. (5)

4. Answer the following:

(a) Show that any infinite subset of a countable set is countable. (5)

(b) If every element of a group $(G, *)$ be its own inverse, then show that $(G, *)$ is abelian. (5)

(c) Prove that every finite integral domain is a field. (5)

5. Answer the following:

(a) Represent the argument symbolically and determine whether the argument is valid.

If it rains today, then we will not have a party today.

If we do not have party today, then we will have a party tomorrow.

Therefore, if it rains today, then we will have a party tomorrow. (5)

(b) Obtain the principal disjunctive normal form of $\sim p \vee q$. (5)

(c) Solve the recurrence relation $a_n = a_{n-1} + 2a_{n-2}, n \geq 2$ with initial conditions $a_0 = 0, a_1 = 1$. (5)

6. Answer the following :

- (a) Use generating functions to solve the recurrence relation $\alpha_n - 9\alpha_{n-1} + 20\alpha_{n-2} = 0$ with initial condition $\alpha_0 = -3, \alpha_1 = -10$. (5)
- (b) Find the generating function in closed form of the sequences 1,1,0,1,1,1,1,... (5)
- (c) Prove that the following propositions are tautology. (2+3=5)
- (i) $p \vee \sim p$
- (ii) $(p \wedge q) \rightarrow p$.

7. Answer the following:

- (a) Find the GCD of 595 and 252 and express it in the form $252m + 595n$. (5)
- (b) If a, b, c, d are integers such that $a \equiv b \pmod{m}$ and $c \equiv d \pmod{m}$, then show that $a - c \equiv b - d \pmod{m}$. (5)
- (c) If $U = \{1,2,3,4,5,6,7,8,9,10\}$, $A = \{3,4,5\}$ and $B = \{1,3,6,10\}$, then find the string representation for A and B . (5)

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