

Total No. of printed pages = 4

MA 171301

Roll No. of candidate

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15/2/ 2021

B.Tech. 3rd Semester End-Term Examination

All

MATHEMATICS – III

(New Regulation)

Full Marks – 70

Time – Three hours

The figures in the margin indicate full marks for the questions.

Answer question No. 1 and any four from the rest.

1. Choose the correct answers : (10 × 1 = 10)

(i) The solution of the partial differential equation $pq = k$ is

(a) $z = ax + \frac{k}{a}y + c$

(b) $z = ax + c$

(c) $z = by + c$

(d) $z = k(x + y)$

(ii) The order and degree of the partial differential equation

$$\frac{\partial^3 z}{\partial x^3} + \left(\frac{\partial^2 z}{\partial x^2}\right)^2 + \left(\frac{\partial z}{\partial x}\right)^4 = 0$$
 is

(a) 1,3

(b) 3,4

(c) 1,4

(d) 2,4

(iii) The function $u = e^x \cos y$ is

(a) non-harmonic

(b) harmonic

(c) non-analytic

(d) analytic

(iv) For a complex function $f(z) = u + iv$, where $u = u(r, \theta)$, $v = v(r, \theta)$

(a) $\frac{\partial u}{\partial r} = \frac{\partial v}{\partial \theta}$

(b) $\frac{\partial u}{\partial r} = \frac{1}{r} \frac{\partial v}{\partial \theta}$

(c) $\frac{\partial u}{\partial \theta} = \frac{\partial v}{\partial r}$

(d) $\frac{\partial v}{\partial \theta} = \frac{1}{r} \frac{\partial v}{\partial r}$

[Turn over

- (v) If $f(z)$ is analytic and $f'(z)$ is continuous at each point within and on a simple closed curve C , then $\oint_C f(z) dz =$
- (a) 0 (b) $f'(z)$
(c) $2\pi i$ (d) $\frac{1}{2\pi i}$
- (vi) If A and B are any two mutually exclusive events in the sample space S , then
- (a) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
(b) $P(A \cup B) = P(A) + P(B)$
(c) $P(A \cup B) = P(A) + P(B) - P(A \setminus B)$
(d) $P(A \cup B) = P(A) + P(B) - P(B \setminus A)$
- (vii) The area under the normal curve is
- (a) 1 (b) 0.5
(c) 0.25 (d) 1.5
- (viii) Which of the following is an odd function?
- (a) $x^4 + 5x^2$ (b) $x^4 + 5x$
(c) $x^3 + 5x$ (d) $x^3 + 5x^4$
- (ix) $\cos x$ is a periodic function with period
- (a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$
(c) π (d) 2π
- (x) The standard deviation of the binomial distribution is
- (a) npq (b) \sqrt{npq}
(c) \sqrt{np} (d) pq
2. (a) Form a partial differential equation by eliminating the arbitrary function f from $z = x^n f\left(\frac{y}{x}\right)$. (4)
- (b) Solve $(y - z)p + (z - x)q = x - y$. (6)
- (c) Solve $(px + qy - z)^2 = p^2 + q^2$. (5)

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3. (a) Using Charpit's method solve the equation $qx + py = pq$. (5)
- (b) A string is fastened to two points l apart. Motion is started by displacing the string in the form $y = a \sin \frac{\pi x}{l}$ from which it is released at time $t = 0$. Show that the displacement of any point at a distance x from one end at a time t is given by $y(x, t) = a \sin \frac{\pi x}{l} \cos \frac{\pi ct}{l}$. (10)
4. (a) Prove that the function $f(z) = e^x(\cos y + i \sin y)$ is analytic and find its derivative. (3+2 = 5)
- (b) Determine the analytic function whose real part is given by $u = 2x(1 - y)$. (4)
- (c) Under the transformation $w = \frac{1}{z}$ find the image of $|z - zi| = 2$. (6)
5. (a) Evaluate $\int_0^{1+i} (x^2 - iy) dz$ along the path $y = x^2$. (5)
- (b) State Cauchy's integral formula. Using Cauchy's integral formula, evaluate $\oint_C \frac{\cos z}{z - \pi} dz$ where C is the circle $|z - 1| = 3$. (2+5 = 7)
- (c) Expand $f(z) = \frac{1}{(z+1)(z+3)}$ in Taylor's series for the region $|z| < 1$. (3)
6. (a) How many tosses of a fair coin are needed so that the probability of getting at least one head is 87.5%? (3)
- (b) In a certain college 4% of boys and 1% of girls are taller than 1.8 m. Furthermore, 60% of the students are girls. If a student is selected at random and is found taller than 1.8 m. What is the probability that the student is a girl? (5)
- (c) Define binomial distribution and find its mean. (7)
7. (a) If X is a normally distributed variable with mean $\mu = 30$ and standard deviation $\sigma = 4$, find
- (i) $P(X < 40)$
- (ii) $P(X > 21)$
- (iii) $P(30 < X < 35)$ (10)
- (b) Ten persons among whom are A and B, sit down at a round table. Find the probability that there are three persons between A and B. (5)

8. (a) Find Fourier series expansion of the function $f(x) = x$, $0 < x < 2\pi$. (6)

(b) Find the Fourier series for the function $f(x) = x - x^2$, $-\pi < x < \pi$. Also deduce $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots = \frac{\pi^2}{12}$. (7+2 = 9)