MA 181301 B

Roll No. of candidate

18/2/   2021

B.Tech. 3rd Semester End-Term Examination

MATHEMATICS – III B

Full Marks – 70   Time – Three hours

The figures in the margin indicate full marks

for the questions.

Answer question No. 1 and any *four* from the rest.

1. (a) Fill in the blanks : $(8 \times 1 = 8)$

     (i)   Kurtosis measures the —————— of a distribution.

     (ii) If the two regression coefficients are –0.4 and –0.9 respectively, then the correlation coefficient is ——————

     (iii) If $f(x) = \dfrac{1}{2}(x + 1)$ for $-1 < x < 1$ and 0 otherwise, represents the density function of a random variable x, then $E(x) =$ ——————.

     (iv) If x is a Poisson variable such that $P(x = 1) = 0.3$ and $P(x = 2) = 0.2$, then $P(0) =$ ——————.

     (v)   If A and B are events such that $P(A \cup B) = \dfrac{3}{4}$, $P(A \cap B) = \dfrac{1}{4}$

           $P(\overline{A}) = \dfrac{2}{3}$, then $P(A) =$ ——————.

     (vi) If the two regression lines are $3x + 2y = 26$ and $6x + y = 31$, then $\bar{x}$ and $\bar{y}$ are respectively.

     (vii) A group of 100 items have a mean of 60. If the mean of 60 of these items be 51, then the mean of the other 40 items is ——————.

     (viii) The first three moments of a distribution about the value 3 are –1, 10, –28. The third moment about the mean is ——————.

[Turn over

(b) Choose the correct answer : $(2 \times 1 = 2)$

(i) Which of the vectors is a probability vector?

(1) $\left(\dfrac{1}{4}, \dfrac{1}{2}, 0, \dfrac{1}{4}\right)$ (2) $\left(\dfrac{1}{3}, \dfrac{2}{3}, -\dfrac{1}{4}, \dfrac{1}{4}\right)$

(3) $\left(\dfrac{1}{2}, \dfrac{1}{3}, -\dfrac{1}{5}, \dfrac{1}{7}\right)$ (4) $(3, 4, 5, 0)$

(ii) If $f(x) = x + \dfrac{2}{k}$, $x = 1, 2, 3, 4, 5$ is the probability function of a discrete random variable, then $k = ?$

(1) $\dfrac{5}{7}$ (2) $\dfrac{7}{5}$

(3) $-\dfrac{5}{7}$ (4) $-\dfrac{7}{5}$

2. (a) The scores of two golfers for 10 rounds each are given below :

    A: 58   59   60   54   65   66   52   75   69   52

    B: 84   56   92   65   86   78   44   54   78   68

Which may be regarded as the more consistent player?   (6)

(b) Find the coefficient of correlation for the following data and discuss the nature of correlation. $(4+1=5)$

$x:$ 1   2   3   4   5

$y:$ 10   12   16   27   25

(c) A committee consists of 8 students two of which are from 1st Year, three from 2nd year and three from 3rd year. What is the chance that

(i) the three students belong to different classes?

(ii) two belong to the same class and the 3rd to different class? $(2+2=4)$

3. (a) A letter is known to come either from London or from Clifton. On the post only the consecutive letters on are legible. Find the probability that it came from London.   (4)

(b) Fit a least square straight line to the following data: $(4+1=5)$

$x:$ 2   7   9   1   5   12

$y:$ 13   21   23   14   15   21

Hence find $y(10)$.

(c) Calculate the moment coefficient of skewness and kurtosis of the following data :

Class:	0-4	4-8	8-12	12-16	16-20
Frequency:	4	10	6	12	8

Hence comment on their nature. $(4+2=6)$

4. (a) From the following data obtain the two regression lines and hence find the correlation coefficient. $(2+2+2=6)$

x: 100 98 78 85 110 93 80

y: 85 90 70 72 95 81 74

   (b) A machine produces an average of 20% defective bolts. A batch is accepted if a sample of 5 bolts taken from that batch contains no defective and rejected if it contains 3 or more defective. In other cases a second sample is taken. What is the probability that the second sample is required?   (4)

   (c) Find the mean and standard deviation of a normal distribution in which 5% of the items are under 30 and 80% are under 50.   (5)

5. (a) Under what conditions Poisson distribution is a limiting case of bionomial distribution? Verify it.   (4)

   (b) A sample of six fathers and their eldest sons gave the following data about their heights in inches.

Father (X) 65 63 67 64 68 62

Son (Y)   68 60 68 65 69 61

Calculate the coefficient of rank correlation.   (5)

   (c) Write one use of t-test. A filling machine is expected to fill 5 kg of powder into bags. A sample of 10 bags gave the following weights 4.7, 4.9, 5.0, 5.1, 5.4, 5.2, 4.6, 5.1, 4.6 and 4.7. Test whether the machine is working properly or not. $(1+5=6)$

6. (a) Define null hypothesis in test of significance. A sample analysis of examination results of 600 students, it was found that 280 students have failed, 170 have secured a 3rd class, 90 have secured a second class and the rest a first class. Do this data supports the general belief that above categories are in the ratio 4:3:2:1 respectively. $(1+4=5)$

   (b) Two dice are thrown simultaneously. Let X be the random variable denoting the sum of the two faces obtained. Write the distribution of X and find the mean. $(3+1=4)$

   (c) Determine

      (i) marginal distributions of x and y

      (ii) $E(x)$, $E(y)$ and $E(x, y)$ and

      (iii) $h(1/y = 1)$ for the following joint probability distribution. $(2+3+1=6)$

x/y	1	2	3
1	1/12	1/6	0
2	0	1/9	1/5
3	1/18	1/4	2/15

7. (a) Find the unique fixed probability vector for the matrix $A = \begin{pmatrix} 0 & 1 & 0 \\ \frac{2}{3} & 0 & \frac{1}{3} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix}$. (4)

(b) Define regular stochastic matrix. Test whether the matrix $A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} \end{pmatrix}$ is

regular stochastic or not. (1+4=5)

(c) The diameter of an electric cable is assumed to be a continuous random variable x with probability density function $f(x) = 6x(1-x),\ 0 \le x \le 1$. Determine b such that $P[x < b] = P[x > b]$. (4)

(d) Write the transition matrix for the following diagram. (2)