Total No. of printed pages = 4

MA 131501 NR

Roll No. of candidate

17/2/22 **2021**

B.Tech. 5th Semester End-Term Examination

CSE

PROBABILITY AND RANDOM PROCESS

(New Regulation)

Full Marks – 70 Time – Three hours

The figures in the margin indicate full marks for the questions.

Answer Question No.1 and any *four* from the rest.

1. Choose the correct answer : $(10 \times 1 = 10)$

   (i) $P(A \cap B) + P(A \cap \overline{B}) =$

   (a) $P(A)$ (b) $P(A \cup B)$

   (c) $P(B)$ (d) $P(A \cap B)$

   (ii) If X is a continuous random variable, then find out the correct option from the following

   (a) $F_X(x) = \frac{d}{dx} f_X(x)$ (b) $F_X(x) = \int_{-\infty}^{x} f_X(x)dx$

   (c) $F_X(x) = \int_{-\infty}^{0} f_X(x)dx$ (d) $f_T(x) = \frac{d}{dx} F_X(x)$

   (iii) The mean of a uniformly distributed rancom variable X within an interval $[0, 2]$ is

   (a) 0 (b) 1

   (c) 2 (d) 3

[Turn over

(iv) If $f_{X,Y}(x, y)$ is the joint probability density function of X and Y, the marginal probability density function of Y is

(a) $\int_{-\infty}^{\infty} f_{X,Y}(x,y)dy$ (b) $\int_{-\infty}^{\infty} f_{X,Y}(x,y)dx$

(c) $\int_{0}^{\infty} f_{X,Y}(x, y)dx$ (d) $\int_{0}^{\infty} f_{X,Y}(x,y)dy$

(v) Two continuous random variable X and Y are said to be independent if

(a) $f_{X,Y}(x,y) = f_X(x)f_Y(y)$ (b) $p_{X,Y}(x,y) = p_X(x)p_Y(y)$

(c) $p_X(x) = p_{X,Y}(x,y)p_Y(y)$ (d) $f_Y(y) = f_X(x)f_{X,Y}(x,y)$

(vi) The area under normal curve between $Z = -\infty$ and $Z = \infty$ is

(a) 1 (b) 0

(c) 0.5 (d) 0.05

(vii) If $X(t)$ is differentiable, then $-\frac{d^2}{d\tau^2}[R_{XX}(\tau)]$ is

(a) $R_{XX}(\tau)$ (b) $[R_{XX}(\tau)]^2$

(c) $E[X^2(t)]$ (d) $E[X(t)]$

(viii) The average power in the frequency band $[w_1, w_2]$ is

(a) $\frac{1}{2\pi}\int_{w_1}^{w_2} S_{XX}(\omega)d\omega$ (b) $\frac{1}{\pi}\int_{w_1}^{w_2} S_{XX}(\omega)d\omega$

(c) $\frac{2}{\pi}\int_{w_1}^{w_2} S_{XX}(\omega)d\omega$ (d) $\frac{\pi}{2}\int_{w_1}^{w_2} S_{XX}(\omega)d\omega$

(ix) If the probability density function of continuous random variable X is given by $f_X(y) = \begin{cases} \frac{10}{x^2}, & x > 10 \\ 0, & x \leq 10 \end{cases}$, then the value of $P(X > 20)$ is

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$

(c) $\frac{1}{8}$ (d) 1

(x) The variance of an exponential distribution with parameter $\lambda > 0$ is

(a) $\frac{1}{\lambda}$ (b) λ

(c) $\frac{1}{\lambda^2}$ (d) λ^2

2. Answer the following :

(a) Prove that the moment generating function of the sum of two independent random variables is the product of their generating function. (5)

(b) The joint probability density function of random variables X and Y is given by $f_{X,Y}(x,y) = \frac{1}{4}e^{-|x|-|y|}$ for $0 < x < \infty$ and $0 < y < \infty$. Show that X and Y are independent. (5)

(c) The number of years a washing machine functions is a exponentially distributed with $\lambda = \frac{1}{10}$. What is the probability that it will be working after an additional 10 years? (5)

3. Answer the following :

(a) Given the random process $X(t) = A\sin(\omega t + \theta)$, where A and w are constants and θ is uniformly distributed random variable in $[-\pi, \pi]$.

(i) Define a new random process $Y(t) = X^2(t)$.

(ii) Are $X(t)$ and $Y(t)$ wide-sense stationary? (1 + 4 = 5)

(b) If X be a random variable non-negative values then prove that $P[X \geq a] \leq \frac{E(X)}{a}$ for any $a > 0$. (5)

(c) Consider two random variables X and Y are independent, zero mean with variance 49 and 25 respectively, Find the correlation coefficient between $X - Y$ and $X - Y$. (5)

4. Answer the following :

(a) A random variable X has a characteristic function given by

$$\phi_X(\omega) = \begin{cases} 1 - |w| & , |w| \leq 1 \\ 0 & , |w| > 1 \end{cases}$$

Find the probability density function of X. (5)

(b) Find the variance and autocorrelation function of the random process $X(t) = tU$, where U is a uniformly distributed random variable in [0, 1]. (5)

(c) For a random process with power spectrum $S_{XX}(\omega) = \frac{w^2}{w^4 + 10w^2 + 9}$, find the autocorrelation function of the process. (5)

5. Answer the following :

(a) The autocorrelation function of a stationary random process is $R_{XX}(t) = 16 + \dfrac{2}{1+2\tau^2}$. Find the mean and variance of the process. (5)

(b) Let the probability density function of a random variable X is $f_X(x) = \dfrac{1}{2}e^{-|x|}$, $-\infty < x < \infty$. Find the cumulative distribution function of X. (5)

(c) Let X be independent Poisson random variable with parameter λ. Show that $P(X \text{ is even}) = \dfrac{1}{2}\left(1+e^{-2\lambda}\right)$. (5)

6. Answer the following :

(a) A bag contains 4 white, 6 red and 8 black balls. Four balls are drawn one by one with replacement. What is the probability that at least one is white? (5)

(b) The average test marks in a class is 80. The standard deviation is 6. If the marks are distributed normally, how many students in a class of 200 receive marks between 70 and 80. (5)

(c) If X is uniformly distributed in $(-1, 3)$, find the probability density function of $Y = X^2$. (5)

7. Answer the following :

(a) Find the rms bandwidth for the power spectrum $S_{XY}(\omega) = \dfrac{1}{\left[1+\left(\dfrac{\omega}{W}\right)^2\right]^2}$. (5)

(b) Determine which of the following systems are linear. (5)

(i) $y(t) = |x(t)|$

(ii) $y(t) = tx(t)$

(c) The joint probability mass function of two random variables X and Y is

$$p_{XY}(x,y) = \begin{cases} k(2x+y); & x = 1, 2;\ y = 1, 2 \\ 0 & \text{otherwise} \end{cases}$$

(i) Find the value of k

(ii) Find the marginal probability mass functions of X and Y. (2 + 3 = 5)