Total No. of printed pages = 4

MA 181301 A

Roll No. of candidate | | | | | | | | | | |

18/21 **2021**

B.Tech. 3rd Semester End-Term Examination

CE, ME, EE, CHEE, PEIE, IE, IPE, EEE

MATHEMATICS – III A

(New Regulation & New Syllabus)

Full Marks – 70 Time – Three hours

The figures in the margin indicate full marks for the questions.

Answer question No. 1 and any *four* from the rest.

1. Answer the following: $(10 \times 1 = 10)$

 (a) The differential equation of all spheres whose centre lies on the z-axis is ______.

 (b) Solution of $\frac{\partial^2 z}{\partial x^2} = xy$ is ______.

 (c) The complete solution of the different equation $z = px + qy + f(p, q)$ is ______.

 (d) If $L\{f(t)\} = \bar{f}(s)$, then $L\{f(at)\} =$ ______.

 (e) If $L^{-1}\{\bar{f}(s)\} = f(t)$, then $L^{-1}\left\{\frac{1}{s}\bar{f}(s)\right\} =$ ______.

 (f) The probability that A passes a test is $\frac{2}{3}$ and that B passes the same test is $\frac{3}{5}$, the probability (i) $\frac{2}{5}$, (ii) $\frac{4}{15}$, (iii) $\frac{2}{15}$, (iv) $\frac{7}{15}$.

 (g) If $f(x) = kx^3$, $0 < x < 1$ and 0 elsewhere is a p.d.f. then value of k is (i) 1, (ii) 4, (iii) $\frac{1}{4}$, (iv) none of these

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(h) If X is a Poisson variate such that $P(X=2)=P(X=3)$, then $P(X=0)$ is __________.

(i) The main use of χ^2 - test is __________.

(j) A stochastic matrix P is said to be regular if __________.

2. Answer the following

(a) Derive a partial differential equation by eliminating arbitrary functions from $z = yf(x) + xg(y)$. (3)

(b) Solve the following

(i) $x^2p^2 + y^2q^2 = z^2$. (5)

(ii) $zpq = p + q$. (3)

(iii) $(z^2 - 2yz - y^2)p + (xy + xz)q = xy - xz$. (4)

3. Answer the following

(a) Determine the solution of one dimensional heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ where the boundary conditions are $u(0,t) = 0 = u(l,t)(t > 0)$ and the initial condition is $u(x,0) = x, l$ being the length of the bar. (7)

(b) X is a continuous random variable with probability density function given by $f(x) = x^3, \quad 0 \le x \le 1$
$\qquad = (2-x)^3, \quad 1 \le x \le 2$

Calculate the mean and the standard deviation of the distribution. (2+3 = 5)

(c) Find the Laplace transform of $\{te^{-2t}\sin t\}$. (3)

4. Answer the following

(a) In a certain collage 4% of the boys and 1% of the girls are taller than 1.8 m. Furthermore 60% of the students are boys. If a student is selected at random and is found to be taller than 1.8 m, what is the probability that the student is a girl? (4)

(b) The probability that a bomb dropped from a plane will strike the target is $\frac{1}{5}$. If six bombs are dropped, find the probability that (i) exactly three (ii) at least two will strive the target. (2+3 = 5)

(c) A source of liquid is known to certain bacteria with the mean number of bacteria per cc equal to 3. Ten 1 cc test tubes are filled wit the liquid. Using Poisson distribution, calculate the probability that all the test tubes will show growth, i.e. at least 1 bacteria is contained in each. (3)

(d) Find $L\left\{\frac{1-\cos t}{t^2}\right\}$. (3)

5. Answer the following

(a) Solve $(p^2+q^2)y = qz$ by Charpit's method. (6)

(b) The income of a group of 10,000 persons was found to be normally distributed with mean Rs. 750 pm and standard deviation of Rs. 50. Find (i) What percentage of people had income more than Rs. 668 p.m., (ii) the lowest income among the richest 100. (2+3 = 5)

(c) Five dice were thrown 96 times and the number of times 4, 5 or 6 were thrown were:

No. of dice shown 4, 5 or 6:	5	4	3	2	1	0
Frequency:	8	18	35	24	10	1

Find the probability of a getting this result by chances. (4)

6. Answer the following

(a) (i) Find $L^{-1}\left\{\frac{1}{s^2(s^2+a^2)}\right\}$.

(ii) Using convolution theorem find $L^{-1}\left\{\frac{1}{(s+1)(s+2)}\right\}$. (3+3)

(b) A random sample of 8 boys had the following I.Q. 70 120 110 101 88 86 95 98. Do these data support the assumption of a population mean I.Q. of 100? (6)

(c) Fit a straight line to the following data. (3)

Year x:	1961	1971	1981	1991	2001
Production y:	8	10	12	10	16

(in thousand tones)

7. (a) (i) Solve by using Laplace transform $\frac{d^2x}{dt^2}+2\frac{dx}{dt}+5x = e^{-t}\sin t$, $x(0)=0$, $x'(0)=1$. (7)

Or

(ii) A tightly stretched flexible string has its ends fixed at $x = 0$ and $x = l$. At time $t = 0$ the string is given a shape defined by $f(x) - \mu x(l - x)$, where μ is a constant and then released. Find the displacement of any point x of the string at any time $t > 0$. (7)

(b) Give the joint distribution

y \ x	0	1	2
0	0.1	0.4	0.1
1	0.2	0.2	0

Determine (i) marginal distribution of X and Y (ii) $E(X)$, $E(XY)$ and $h(x/_y = 1)$. (5)

(c) Write the transition matrix for the transition diagram. (3)