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MA 171301

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Roll No. of candidate | | | | | | | | | | |

2018

B.Tech. 3rd Semester End-Term Examination

MATHEMATICS — III

(New Regulation)

(w.e.f. 2017-2018)

Full Marks – 70 Time – Three hours

The figures in the margin indicate full marks for the questions.

Answer question No. 1 and any *four* from the rest.

1. In each of the following questions, four answers are provided of which only one is correct. Choose the correct answer. (10 × 1 = 10)

   (i) The order of the partial differential equation $\left(\frac{\partial Z}{\partial x}\right)^2 + \frac{\partial^3 Z}{\partial y^3} = z\frac{\partial Z}{\partial x}$ is

   (a) 0 (b) 1

   (c) 2 (d) 3

   (ii) The solution of the partial differential equation $pq = k$ is

   (a) $Z = ax + \frac{k}{a}y + c$

   (b) $Z = ax + c$

   (c) $Z = by + c$

   (d) $Z = k(x + y)$

[Turn over

(iii) The function $u = e^x xos\, y$ is

(a) non–harmonic

(b) harmonic

(c) non–analytic

(d) none of these

(iv) (a) The real part of an analytic function is harmonic, whereas the imaginary part is not harmonic

(b) The real part of an analytic function is not harmonic, whereas the imaginary part is harmonic

(c) The real part as well as the imaginary part of an analytic function is harmonic

(d) Neither the real part nor the imaginary part of an analytic function is harmonic

(v) Let A, B be two events such that $P(A \cup B) = \frac{1}{2}$, $P(A) = \frac{2}{3}$, $P(B) = \frac{2}{5}$. Then $P(A \cap B)$ is

(a) $\frac{1}{3}$ (b) $\frac{3}{5}$

(c) $\frac{13}{30}$ (d) $\frac{17}{30}$

(vi) Binomial distribution has

(a) 1 (b) 2

(c) 3 (d) 4

(vii) The area under the normal curve is

(a) 0.25 (b) 0.5

(c) 0.75 (d) 1

(viii) sin x function is a periodic function with period

(a) 2π (b) π

(c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$

(ix) Which of the following is an even function of x ?

(a) $x^4 + 5x^2$ (b) $x^4 + 5x$

(c) $x^3 + 5x$ (d) $x^3 + 5x^2$

(x) Fourier coefficient a_0 in the Fourier series

$\frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$ of $f(x) = e^{-x}$, $0 < x < 2\pi$, and $f(x+2\pi) = f(x)$ is

(a) $\frac{1}{\pi}(e^{-2\pi} - 1)$ (b) $\frac{1}{2\pi}(1 - e^{-2\pi})$

(c) $\frac{1}{\pi}(1 - e^{-2\pi})$ (d) None of these

2. (a) Form a partial differential equation by eliminating the arbitrary functions f, and g from $Z = f(x+at) + g(x-at)$. (3)

(b) Solve the partial differential equation by Lagrange's method.

$p \tan x + q \tan y = \tan Z$, where p, and q have their usual meanings. (3)

(c) Solve the partial differential equation $Z = p^2 + q^2$,

where p, and q have their usual meanings. (4)

(d) Solve the partial differential equation

$$\frac{\partial^2 Z}{\partial x^2} - 2\frac{\partial^2 Z}{\partial z \partial y} + \frac{\partial^2 Z}{\partial y^2} = e^{2x+3y}. \quad (5)$$

3. (a) Solve the partial differential equation

$$\frac{\partial^2 Z}{\partial x^2} - \frac{\partial^2 Z}{\partial y^2} + \frac{\partial Z}{\partial x} - \frac{\partial Z}{\partial y} = 0. \quad (5)$$

(b) Solve the Laplace equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0. \quad (10)$$

Which satisfies the conditions

$$u(0, y) = u(l, y) = u(x, 0),\ u(x, a) = \sin \frac{n\pi x}{l}.$$

4. (a) Prove that the function

$f(Z) = e^x (\cos y + i \sin y)$

is analytic, and find its derivative. (5)

(b) Determine the region in the W-plane in which the rectangle branded by the lines $x = 0$, $y = 0$, $x = 2$, and $y = 1$ is mapped under the transformation. (5)

$W = Z + (2 + 3i)$

(c) Evaluate $\oint_C |Z|^2 dz$ around the square C with vertices $(0,0)$, $(1, 0)$, $(1, 1)$, and $(0, 1)$. (5)

5. (a) In a certain college 4% of boys, and 1% of girls are taller than 1.8 m. Furthermore, 60% of the students are girls. If a student is selected at random and is found taller than 1.8 m. What is the probability that the student is a girl? (5)

(b) An unbiased coin is tossed six times. What is the probability that the tosses will result in at least five heads? (5)

(c) One percent of bulbs manufactured by a firm are expected to be defective. A carton contains 200 bulbs. Find the probability that the carton contains 3 or were defective bulbs. (5)

6. (a) State Cauchy's integral formula. Using Cauchy's integral formula, evaluate $\oint_C \frac{Z^2+5}{Z-3} dZ$, where C is the circle $|Z| = 4$. (5)

(b) Evaluate the following integral by using Residue theorem.

$\oint_C \frac{Z}{(Z-1)\,(Z-2)^2} dZ$, where C is the circle $|Z-2| = \frac{1}{2}$. (5)

(c) Following table gives the number of train accidents in a country that occured during the various days of the week. Find whether the accidents are uniformly distributed over the week. (5)

Days	Sun	Mon	Tues	Wed	Thurs	Fri	Sat
No. of accidents	20	18	13	23	26	11	15

7. (a) State Dirichlet's conditions for a Fourier series. (4)

(b) Find Fourier series expansion of the function $f(x) = x,\ 0 < x < 2\pi$. (5)

(c) Find the half-range sine series for the function

$$f(x)\begin{cases} = x, & 0 < x < \dfrac{\pi}{2} \\ = \pi - x, & \dfrac{\pi}{2} < x < \pi \end{cases}$$ (6)
