Total No. of printed pages = 4

BCA 171104

Roll No. of candidate | | | | | | | | | | |

23131 **2022**

B.C.A. 1st Semester End-Term Examination

MATHEMATICS – I

Full Marks – 70 Time – Three hours

The figures in the margin indicate full marks for the questions.

Answer question No. 1 and any *four* from the rest.

1. In each of the following questions four answers are provided of which only one is correct. Choose the correct answer : $(10 \times 1 = 10)$

(i) The diagonal elements of a skew symmetric matrix are all

(a) zero (b) one

(c) non zero, but equal (d) unequal real numbers

(ii) The value of the determinant $\begin{bmatrix} 5 & 7 \\ 2 & 3 \end{bmatrix}$ is

(a) 29 (b) -1

(c) 1 (d) 31

(iii) The order of the matrix $A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 9 & 10 & 11 & 12 & 13 & 14 \end{bmatrix}$ is

(a) 6×2 (b) 2×6

(c) 2×2 (d) 6×6

(iv) Let A, B be two matrices such that AB is defined. Let A^t denote the transpose of A. Then

(a) $(AB)^t = A^t B^t$ (b) $(AB)^t = B^t A^t$

(c) $(AB)^t = AB$ (d) $(AB)^t$ does not exists

[Turn over

(v) The sum of the eigen values of the matrix $A = \begin{bmatrix} 4 & 1 \\ 1 & 4 \end{bmatrix}$ is

(a) 15 (b) 10

(c) 2 (d) 8

(vi) $\lim_{x \to 2} \frac{x^2 - 4}{x - 2}$

(a) is 0 (b) is 1

(c) does not exists (d) is 4

(vii) The derivative of x^{10} with respect to x is

(a) $\frac{x^{11}}{11}$ (b) $10x^{10}$

(c) $10x^9$ (d) $9x^9$

(viii) The derivative of a constant

(a) is 0 (b) is 1

(c) does not exists (d) is any non zero number

(ix) The conjugate of $-3+4i$, $(i = \sqrt{-1}$, is

(a) $-3-4i$ (b) 1

(c) $3+4i$ (d) 5

(x) The modulus of $-5+12i$, $(i = \sqrt{-1})$, is

(a) 7 (b) 13

(c) $-5-12i$ (d) $5-12i$

2. (a) Write down two non zero matrices A and B such that AB is a zero matrix. (2)

(b) Write down two non zero matrices A and B such that $AB = BA$. (2)

(c) Express the matrix

$$A = \begin{bmatrix} 2 & 4 & 6 \\ 8 & 10 & 12 \\ 14 & 16 & 18 \end{bmatrix}$$

as the sum of symmetrix and a skew symmetric matrix. (5)

(d) Solve the following system of equations by Cramer's rule. (6)

$2x + y + z = 4$

$x - 2y + 3z = 2$

$2x + 3y - 5z = 0$

3. (a) If the matrix $A = \begin{bmatrix} 3-2x & x+1 \\ 2 & 4 \end{bmatrix}$ is singular, what is the value of x? (2)

(b) If $2\begin{bmatrix} 3 & 4 \\ 5 & x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$, find the values of x and y. (2)

(c) If $A = \begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix}$ and $f(x) = 2x^2 + 4x + 5$, find $f(A)$ (3)

(d) If $A = \begin{bmatrix} 2x & 0 \\ x & x \end{bmatrix}$, and $A^{-1} = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$, find the value of x. (2)

(e) Let $A = \begin{bmatrix} 1 & 3 \\ 4 & 5 \end{bmatrix}$ Find the eigenvalues of A verify Cayley-Hamilton theorem for the matrix A, hence find A^{-1}. (2+2+2=6)

4. (a) If $A = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$, find the product of the eigen values of A. (3)

(b) For the matrix $A = \begin{bmatrix} 1 & -2 & 4 \\ 0 & 2 & 1 \\ -4 & 5 & 3 \end{bmatrix}$, verify that $A(adjA) = (adjA) = |A|I$. (6)

(c) Define rank of a matrix. What is the rank of a null matrix? What is the rank of the matrix $A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 4 & 6 & 8 & 10 & 13 \end{bmatrix}$? (6)

5. (a) Find the values of x and y if $2 + (x + iy) = (3 - i)$, $i = \sqrt{-1}$. (2)

(b) Express $\dfrac{2-3i}{2+3i}$ in the form $a + ib$ $i = \sqrt{-1}$. (3)

(c) If $(a + ib)(c + id) = A + iB$, $i = \sqrt{-1}$, Prove that

(i) $(a - ib)(c - id) = A - iB$

(ii) $(a^2 + b^2)(c^2 + d^2) = A^2 + B^2$ (5)

(d) If $z = 3 + 2i$, $w = 3 - 2i$, find $z^2 + w^2$. (2)

(e) Express $1 + \sqrt{3}i$ in polar form. (3)

6. (a) Evaluate the following limits. (8)

(i) $\lim_{x \to 1} \frac{x^3 - 1}{x - 1}$

(ii) $\lim_{x \to 3} \frac{x^2 - 4x + 3}{x^2 - 2x - 3}$

(iii) $\lim_{x \to 0} \frac{\sqrt{1+x} - 1}{x}$

(iv) $\lim_{x \to 0} \frac{\sin 3x}{5x}$

(b) Find $\frac{dy}{dx}$ if (4)

(i) $y = \frac{3x^3 + 4x^2 - 7}{x^2}$

(ii) $y = (x^2 + 5x + 6)(x^5 + 11)$

(c) If $y = 6x^5 - 4x^4 - 2x^2 + 5x - 9$, find $\frac{dy}{dx}$ at $x = -1$. (3)

7. (a) State Rolle's theorem. (3)

(b) Verify Lagrange's Mean value theorem for $f(x) = x(2 - x)$. (4)

(c) Show that the function $f(x) = x^3 - 6x^2 + 12x - 18$ is an increasing function on the set of real numbers. (4)

(d) Find the points of local maxima and local minima and the corresponding local maximum and minimum values of the function $f(x) = -x^3 + 12x^2 - 5$. (4)
