Total No. of printed pages = 3

BCA 171104

Roll No. of candidate | | | | | | | | | | |

22/3 1 2021

B.C.A. 1st Semester End-Term Examination

MATHEMATICS – I

(New Regulation)

Full Marks – 70 Time – Three hours

The figures in the margin indicate full marks
for the questions.

Answer question No. 1 and any *four* from the rest.

1. Answer the following questions : (10 × 1 = 10)

(i) If $\begin{bmatrix} 2x & 0 \\ 0 & y-1 \end{bmatrix} = I_2$, then what are the values of x and y?

(ii) If $A^{-1} = \frac{adj(A)}{11}$, what is the value of $|A|$?

(iii) The eigen values of the matrix A are 1, 2 and 3. What is value of $|A|$.

(iv) Find the value of i^{102}.

(v) If α, β are the roots of the equation $ax^2 + bx + c = 0$, $(a \neq 0)$, what is the value of $\alpha + \beta$?

(vi) $\lim_{x \to 0} \frac{\sin x}{x} = ?$

(vii) If $y = \frac{x-1}{x+1}$, find $\frac{dy}{dx}$.

(viii) If $f''(x) < 0$ at $x = a$, what is the maximum value of $f(x)$?

(ix) If $A = \begin{bmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & f \end{bmatrix}$, what are the eigen values of A^2?

(x) If the tangent to the curve $y = f(x)$ at $x = c$ is parallel to the x – axis, then what is value of $f'(c)$?

[Turn over

2. (a) Find the inverse of $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$. (8)

(b) State Cayley-Hamilton theorem. Verify Cayley-Hamilton theorem for the matrix $\begin{bmatrix} 1 & 2 \\ 0 & -5 \end{bmatrix}$. (2 + 5 = 7)

3. (a) Find the eigen values of the matrix $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$. (8)

(b) Show that, $A\ adj\ (A) = |A| I_3$, where $A = \begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ -1 & 2 & 2 \end{bmatrix}$. (7)

4. (a) Prove that $|z_1 z_2| = |z_1||z_2|$, where z_1 and z_2 are any two complex numbers. (8)

(b) Express $z = 1 - i$ in polar form. (7)

5. (a) If α, β are the roots of the equation $ax^2 + bx + c = 0$, $(a \neq 0)$, find (4 + 4 = 8)

(i) $\frac{1}{\alpha} + \frac{1}{\beta}$

(ii) $\alpha^2 - \beta^2$.

(b) If α, β are the roots of the equation $ax^2 + bx + c = 0$, $(a \neq 0)$ form the equation whose roots are $-\alpha, -\beta$. (7)

6. (a) Evaluate : (5 + 5 = 10)

(i) $\lim_{x \to 3} \frac{x^4 - 81}{x - 3}$

(ii) $\lim_{x \to 0} \frac{\sin 2x}{\sin 3x}$

(b) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$, where $y = x^2 - 2\sin x + \cos x$. (5)

7. (a) Find the maximum and minimum values of the function $f(x) = x^3 - 4.5x^2 + 6x + 1$. (8)

(b) State Rolle's theorem and give the graphical interpretation. (2 + 5 = 7)