Total No. of printed pages = 4

BCA 171104

Roll No. of candidate | | | | | | | | | |

2018

B.C.A. 1st Semester End-Term Examination

MATHEMATICS — I

Full Marks – 70 Time – Three hours

The figures in the margin indicate full marks for the questions.

Answer Q.No. 1 and any *four* from the rest.

1. Answer the following : (10 × 1 = 10)

   (i) What is the conjugate of $(1+i)^2$?

   (ii) Write the simplified value of $i^{31} \times i^{62}$.

   (iii) If $(2x-y)+i2y=(5-2i)$, find the values of x and y.

   (iv) If $z=a+ib$, then $z\bar{z}=$?

   (v) For the quadratic equation $2x^2-3x+1=0$, what is the sum of the roots?

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(vi) If $\begin{pmatrix} x-y & 0 \\ 0 & x+y \end{pmatrix} = I_2$, then find the values of x and y.

(vii) $\lim_{x \to 0} \dfrac{x^3 - 2x^2 + x}{x} = ?$

(viii) If $A^2 + 2A + I_2 = 0$, then write the characteristic equation of the matrix A.

(ix) If $2\begin{vmatrix} 3 & 4 \\ 5 & x \end{vmatrix} + \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = \begin{vmatrix} 7 & 0 \\ 10 & 5 \end{vmatrix}$, then find the value of x?

(x) If the matrix $A = \begin{pmatrix} -2x & x+1 \\ 2 & 4 \end{pmatrix}$ is singular, then $x = ?$

2. (a) Define adjoint of a matrix. Find the adjoint of $A = \begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix}$. (4)

(b) For the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 1 \\ 5 & 2 & 6 \end{bmatrix}$, prove that $A \, adj \, A = |A|$. (5)

(c) Find the inverse of the matrix $A = \begin{pmatrix} 1 & 0 & -2 \\ 2 & 3 & -5 \\ 4 & 6 & 1 \end{pmatrix}$. (6)

3. (a) Find the characteristic equation and the Eigen values of the matrix $A = \begin{pmatrix} 1 & 2 & 5 \\ 2 & 0 & 7 \\ 5 & 7 & 4 \end{pmatrix}$. (8)

(b) Show that $A = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}$ satisfies the equation $A^2 - 4A - 5I = 0$, and hence find A^{-1}. (7)

4. (a) State Cayley-Hamilton theorem. Verify Cayley-Hamilton theorem for the matrix $\begin{bmatrix} 0 & 1 \\ 5 & 4 \end{bmatrix}$. (6)

(b) Solve the system of equations by Cramer's rule. (9)

$$2x - 4y + z = 7$$
$$x - y + 2z = 6$$
$$x + 2y + 3z = 11.$$

5. (a) Express the complex numbers in polar form : (8)

(i) $z = 1 + i$

(ii) $z = 1 - \sqrt{3}\, i$

(b) If $\left| \frac{z - 5i}{z + 5i} \right| = 1$, show that z is a real number. (7)

6. (a) Find all the cube roots of 1. (5)

(b) For any two complex numbers z_1 and z_2, prove that (5 + 5 = 10)

(i) $|z_1 z_2| = |z_1||z_2|$

(ii) $z_1 \bar{z}_1 = |z_1|^2$.

7. (a) Evaluate (any *two*) : (4 × 2 = 8)

(i) $\lim_{x \to 4}\left(\frac{x^2+16}{x+4}\right)$

(ii) $\lim_{x \to 0}\frac{(1+x)^4-1}{x}$

(iii) $\lim_{x \to 0}\left(\frac{\sin ax}{\sin bx}\right)$

(iv) $\lim_{x \to 0}\left(\frac{\tan 3x}{\tan 4x}\right)$

(b) State Lagrange's mean value theorem. (2)

(c) Find the extreme values of the function x^3-6x^2+9x-8 values. (5)