Total No. of printed pages = 4

MA 181102

8/2/23

Roll No. of candidate [| | | | | | | | |]

2023

B.Tech 1st Semester End-Term Examination

MATHEMATICS — I

(New Regulation and New Syllabus)

Full Marks – 70 Time – Three hours

The figures in the margin indicate full marks for the questions.

Answer question No. 1 and any *four* from the rest.

1. Choose the correct answer of each question : $(10 \times 1 = 10)$

(i) If $y = \log(1+x)$, then $y_2(0) =$

(a) 0 (b) 1

(c) $\frac{1}{2}$ (d) -1

(ii) If $u = f\left(\frac{y}{x}\right)$, then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} =$

(a) 0

(b) $f\left(\frac{y}{x}\right)$

(c) $f'\left(\frac{y}{x}\right)$

(d) None of these

(iii) The value of $\Gamma(n)$, where n is a negative integer, is

(a) $n!$

(b) $(n+1)!$

(c) $(n-1)!$

(d) None of these

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(iv) The value of $\int_0^{\frac{\pi}{2}} \sin^7 x dx$ is

(a) $\frac{16}{35}$ (b) $\frac{32}{35}$

(c) $\frac{64}{35}$ (d) $\frac{83}{15}$

(v) If the series $\sum u_n$ is convergent, then

(a) $\lim_{n\to\infty} u_n = 0$

(b) $\lim_{n\to\infty} u_n \neq 0$

(c) $\lim_{n\to\infty} u_n = 1$

(d) $\lim_{n\to\infty} u_n$ does not exist

(vi) Which of the following points is a stationary point of the function $f(x, y) = x^2 + 2x + y^2$

(a) (0, 0)

(b) (0, 1)

(c) (–1, 0)

(d) (1, –1)

(vii) If for a homogenous system of linear equation $AX = 0$ with n number of unknowns, Rank $(A) < n$, then

(a) The system has trivial solution

(b) The system has nontrivial solutions

(c) Both (a) and (b) are correct

(d) Neither (a) nor (b) is correct

(viii) The area enclosed by the curve $y = f(x)$, the x-axis and the ordinates $x = a$ and $x = b$ is given by

(a) $\int_a^b y dx$

(b) $\int_a^b y dy$

(c) $\int_a^b x dx$

(d) $\int_a^b x dy$

(ix) An example for a function which is neither even nor odd is

(a) $x \sin x$

(b) e^{ax}

(c) $\cos x$

(d) none of these

(x) If a function $f(x)$ is periodic with period T, then $f(ax)$, where $a \neq 0$, is also periodic with period

(a) aT

(b) $\frac{T}{a}$

(c) T

(d) $a+T$

2. (a) Conduct a suitable test to discuss the convergence of the series $\sum \frac{n!}{n^n}$. (4)

(b) Apply Leibnitz's rule of successive differentiation to prove the result $(1-x^2)y_{n+2}-(2n+1)xy_{n+1}-n^2y_n=0$, where $y=\sin^{-1}x$. (4)

(c) State L'Hospital's rule. Hence evaluate $\lim_{x\to\frac{\pi}{2}}(\sec x-\tan x)$. (1 + 3 = 4)

(d) Show that a set of mutually orthogonal vectors is linearly independent. (3)

3. (a) State Euler's theorem on homogeneous functions of two variables. Apply Euler's theorem to prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\sin 2u$, if $u=\tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$. (1 + 4 = 5)

(b) Find the minimum value of $x^2+y^2+z^2$ subject to the condition $x+y+z=12$. (4)

(c) If $u=\log(x^3+y^3+z^3-3xyz)$, show that $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=\frac{3}{x+y+z}$. (3)

(d) Expand $f(x)=\sec x$ in powers of x. (3)

4. (a) Find the rank of the matrix $\begin{bmatrix}2 & -2 & 0 & 6\\ 4 & 2 & 0 & 2\\ 1 & -1 & 0 & 3\\ 1 & -2 & 1 & 2\end{bmatrix}$ by reducing to normal form. (5)

(b) Solve the equations by Gauss elimination method (5)

$x+2y+3z=3$

$2x+3y+8z=4$

$5x+8y+19z=11$

(c) Determine the value of b for which the equations (5)

$$2x+y+2z=0$$
$$x+y+3z=0$$
$$4x+3y+bz=0$$

has (i) trivial solution (ii) non trivial solutions.

5. (a) Examine if $A=\begin{bmatrix}-2 & 2 & -3\\ 2 & 1 & -6\\ -1 & -2 & 0\end{bmatrix}$ is diagonalizable. If so find the spectral matrix P and diagonal matrix D, such that $P^{-1}AP=D$. (6)

(b) State Cayley-Hamilton theorem and verify the same for the matrix $A=\begin{bmatrix}3 & 1\\ 2 & 2\end{bmatrix}$. Hence find A^{-1}. (5)

(c) Show that the vectors (2, 1, 1), (1, 2, 1), (1, 1, 2) are linearly independent. (4)

6. (a) Find Fourier series expansion for the function $f(x)=x\sin x, -\pi\le x\le\pi$. Hence deduce that $\frac{\pi-2}{4}=\frac{1}{1.3}-\frac{1}{3.5}+\frac{1}{5.7}-\cdots$ (6)

(b) Find the half range since series for the function $f(x)=\begin{cases}x, & 0<x<\frac{\pi}{2}\\ \pi-x, & \frac{\pi}{2}<x<\pi\end{cases}$ (4)

(c) Evaluate $\int_0^1\int_0^{\sqrt{1-x^2}}\int_0^{\sqrt{1-x^2-y^2}}\frac{dxdydz}{\sqrt{1-x^2-y^2-z^2}}$. (5)

7. (a) Find the radius of curvature of the curve $y=a\log\sec\left(\frac{x}{a}\right)$ at any point (x, y). (3)

(b) Find the volume and surface area of the solid generated by the revolution of the cardioid $r=a(1-\cos\theta)$ about the initial line. (5)

(c) Find the area lying between the parabolas $y^2=4ax$ and $x^2=4ay$. (4)

(d) Using Beta and Gamma function evaluate the integral $\int_0^1 x^4\left(1-\sqrt{x}\right)^5 dx$. (3)
