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MA 181301 B

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Dr. BINA CHOWDHURY, GUWAHATI UNIVERSITY
(GIT & GIPS)
Azara, Haikhowapara,
Guwahati-781017

2023

B.Tech. 3rd Semester End-Term Examination

ECE, ETE, CSE

MATHEMATICS – III (B)

(New Regulation and New Syllabus)

Full Marks – 70

Time – Three hours

The figures in the margin indicate full marks for the questions.

Answer Question No. 1 and any *four* from the rest.

1. Choose the correct answer : (10 × 1 = 10)

(i) The distribution function for a random variable X is $F(x) = 1 - e^{-5x}$, $x \geq 0$.
What is the density function when $x \geq 0$?

- (a) $-5e^{-5x}$ (b) $5e^{-5x}$
(c) e^{-5x} (d) $-5e^{5x}$

(ii) What is the relation between expectation and variance?

- (a) $\sigma^2 = [E(x)]^2 - E(x^2)$
(b) $\sigma^2 = E(x^2) - E(x)$
(c) $\sigma^2 = E(x^2) - [E(x)]^2$
(d) $\sigma^2 = E(x^2) + [E(x)]^2$

(iii) What percent of the values lie outside the area under the normal curve between $x = \mu - 2\sigma$ and $x = \mu + 2\sigma$?

- (a) $4\frac{1}{2}\%$ (b) $4\frac{1}{3}\%$
(c) $2\frac{1}{2}\%$ (d) $2\frac{1}{4}\%$

[Turn over

(iv) Two cards are drawn in succession from a pack of 52 cards. What is the chance that the first card is a king and the second a queen if the first card is not replaced?

(a) $\frac{4}{663}$

(b) $\frac{1}{169}$

(c) $\frac{3}{676}$

(d) $\frac{1}{663}$

(v) What is the value of $\sum_{x=0}^{\infty} P(x)$ where $P(x)$ is a Poisson distribution?

(a) 0

(b) $\frac{1}{2}$

(c) $\frac{1}{4}$

(d) 1

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(vi) Quartile co-efficient of Skewness lies between _____ and _____.

(a) 0 and 1

(b) -1 and 0

(c) -1 and 1

(d) None of these

(vii) The co-efficient of variation where the standard deviation and the mean are 2 and 40 respectively is _____.

(a) 5

(b) 4

(c) 3

(d) 2

(viii) The first two moments of a distribution about the value 5 of the variable are 2 and 20 respectively then the variance is _____.

(a) 18

(b) 16

(c) 24

(d) 22

(ix) Let A and B be two events s.t. $P(A) = \frac{1}{4}$, $P(B) = \frac{1}{3}$ and $P(A \cup B) = \frac{1}{2}$, then

$P(A|B) =$ _____.

(a) $\frac{1}{3}$

(b) $\frac{1}{2}$

(c) $\frac{1}{5}$

(d) $\frac{1}{4}$

(x) The equation of the line of regression of y on x is _____.

(a) $x - \bar{x} = r \frac{\sigma_y}{\sigma_x} (y - \bar{y})$

(b) $x - \bar{x} = r \frac{\sigma_x}{\sigma_y} (y - \bar{y})$

(c) $y - \bar{y} = r \frac{\sigma_x}{\sigma_y} (x - \bar{x})$

(d) None of these

2. Answer the following :

(a) A dice is tossed thrice. A "success" is getting 1 or 6 on a toss. Find the mean and the variance of the number of successes. (5)

(b) A random variable X has the density function $f(x) = \frac{c}{x^2 + 1}$ where $-\infty < x < \infty$.

Find :

(i) The value of c .

(ii) The probability that x^2 lies between $\frac{1}{3}$ and 1. (2 + 3 = 5)

(c) Suppose that the random variables X and Y have a joint density function given by

$$f(x, y) = \begin{cases} c(2x + y); & 2 < x < 6, 0 < y < 5 \\ 0; & \text{otherwise} \end{cases}$$

Find :

(i) The constant c .

(ii) The marginal distribution function for X

(iii) $P(X + Y > 4)$. (1 + 2 + 2 = 5)

3. Answer the following :

(a) The contents of three urns are : 1 white, 2 red, 3 green balls; 2 white, 1 red, 1 green balls and 4 white, 5 red, 3 green balls. Two balls are drawn from an urn chosen at random. These are found to be one white and one green. Find the probability that the balls so drawn came from the third urn. (5)

(b) Five men in a company or twenty are graduates. If 3 men are picked out of 20 at random. What is the probability that

(i) They are all graduates

(ii) At least one is graduate. (2 + 3 = 5)

(c) The probability of a student A passing examination is $\frac{3}{7}$ and of student B passing is $\frac{5}{7}$. Assume the two events "A passes", "B passes" as independent.

Find the probability of

(i) Only A is passing the exam.

(ii) Only one of them is passing the exam. (2 + 3 = 5)

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4. Answer the following :

(a) Find the mean and the variance of Binomial distribution. (2 + 3 = 5)

(b) Fit a Poisson distribution to the following : (5)

$x: 0 \quad 1 \quad 2 \quad 3 \quad 4$

$f: 46 \quad 38 \quad 22 \quad 9 \quad 1$

(c) Assuming that the diameters of 1000 brass plugs taken consecutively from a machine form a normal distribution with mean 0.7515 cm and standard deviation 0.002 cm. How many of the plugs are likely to be rejected if the approved diameter is 0.752 ± 0.004 cm. (5)

5. Answer the following :

(a) Find the mean, median and mode of the following data : (1 + 2 + 2 = 5)

Mid value : 15 20 25 30 35 40 45 50 55

Frequency : 2 22 19 14 3 4 6 1 1

(b) The first four moments of a distribution about the value 3 are 2, 10, 40 and 218 respectively. Find the moments about the origin and mean. (5)

(c) Find the co-efficient of correlation from the following data : (5)

$x: 65 \quad 63 \quad 67 \quad 64 \quad 68 \quad 62 \quad 70 \quad 66$

$y: 68 \quad 66 \quad 68 \quad 65 \quad 69 \quad 66 \quad 68 \quad 65$

6. Answer the following :

(a) If θ is the angle between the two regression lines, Show that

$$\tan \theta = \frac{1-r^2}{r} \cdot \frac{\sigma_x \sigma_y}{\sigma_x^2 + \sigma_y^2}$$

Explain the significance when $r = 0$ and $r = \pm 1$. (3 + 2 = 5)

(b) Write the principles of the method of least square. Obtain the two normal equations to determine the values of a and b for the least square straight line $Y = a + bx$. (4)

(c) Show that $P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$ is a regular stochastic matrix. Also find the

associated unique fixed probability vector. (3 + 3 = 6)

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7. Answer the following :

(a) A set of five similar coins is tossed 320 times and the result is :

No. of heads : 0 1 2 3 4 5

Frequency : 6 27 72 112 71 32

Test the hypothesis that the data follow a Binomial distribution. (5)

(b) A die was thrown 9000 times and a throw of 5 or 6 was obtained 3240 times. On the assumption of random throwing, do the data indicate an unbiased die? (4)

(c) Draw the transition diagram for the following matrix : (2)

$$\begin{matrix} & \begin{matrix} a & b & c \end{matrix} \\ \begin{matrix} a \\ b \\ c \end{matrix} & \left(\begin{array}{ccc} 0.5 & 0.3 & 0.2 \\ 0.1 & 0.7 & 0.2 \\ 0.1 & 0.4 & 0.5 \end{array} \right) \end{matrix}$$

(d) A variate X has the probability distribution :

$$x: -3 \quad 6 \quad 9$$

$$P(x): \frac{1}{6} \quad \frac{1}{2} \quad \frac{1}{3}$$

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Find $E(x)$, $E(x^2)$ and $E(2x+1)^2$.

(1 + 1 + 2 = 4)