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MA 171203

Roll No. of candidate | | | | | | | | | | |

2019

B.Tech. 2nd Semester End-Term Examination

ENGINEERING MATHEMATICS – II

(New Regulation) & New Syllabus (w.e.f. 2017-2018)

Full Marks – 70 Time – Three hours

The figures in the margin indicate full marks for the questions.

Answer Question No. 1 and *four* from the rest.

1. Answer the following : (10 × 1 = 10)

(i) $(AB)^{-1} =$ ————————.

(a) BA

(b) $B^{-1}A^{-1}$

(c) $A^{-1}B^{-1}$

(d) None of the above

(ii) The rank of the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ is

(a) 1 (b) 2

(c) 3 (d) 4

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(iii) A system of equations is said to be inconsistent if it has

(a) No solution

(b) An unique solution

(c) An infinite number of solutions

(d) None of the above

(iv) The dimension of the vector space $\mathbb{R}^3 =$ ________.

(a) 0 (b) 1

(c) 2 (d) 3

(v) The value of $\left[\hat{i}\hat{j}\hat{k}\right] =$ ________.

(a) 1 (b) 2

(c) 3 (d) 4

(vi) The volume of a tetrahedron $ABCD =$ ________.

(a) $\left[\overrightarrow{AB}\,\overrightarrow{AC}\,\overrightarrow{AD}\right]$

(b) $\frac{1}{2}\left[\overrightarrow{AB}\,\overrightarrow{AC}\,\overrightarrow{AD}\right]$

(c) $\frac{1}{3}\left[\overrightarrow{AB}\,\overrightarrow{AC}\,\overrightarrow{AD}\right]$

(d) $\frac{1}{6}\left[\overrightarrow{AB}\,\overrightarrow{AC}\,\overrightarrow{AD}\right]$

(vii) If $\vec{r}$ is the position vector of the point $P(x, y, z)$ w.r.t. origin, then $curl\ \vec{r} =$

(a) 0 (b) 1

(c) 2 (d) None of the above

(viii) Divergence of gradient of a scalar function is equivalent to

(a) Laplacian operator

(b) Curl Opearator

(c) Null vector

(d) None of the above

(ix) $L\{e^{-at}\} =$ ——————.

(a) $\frac{1}{s-a}$

(b) $\frac{1}{s+a}$

(c) $\frac{s}{s-a}$

(d) $\frac{s}{s+a}$, $s > a > 0$

(x) Let $L[f(t)] = F(s)$, then $L[e^{at}f(t)] =$ ——————.

(a) $F(s+a)$

(b) $F(s-a)$

(c) $F(a-s)$

(d) $\frac{1}{s}F(s+a)$

2. (a) Show that for some real number θ, the matrix $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ is orthogonal. (2)

(b) Write $A = \begin{bmatrix} 4 & 5 & 1 \\ 3 & 7 & 2 \\ 1 & 6 & 8 \end{bmatrix}$ as $A = B + C$, where B is symmetric and C is skew-symmetric matrix. (3)

(c) Find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 5 & 7 \end{bmatrix}$ by reducing to echelon form. (5)

(d) Solve the following system of equations by Gauss-Jordan method. (5)

$$x + y + z = 3$$
$$x + 2y + 3z = 4$$
$$x + 4y + 9z = 6.$$

3. (a) Verify Caley-Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$. (5)

(b) Express $v = (1, -2, 5)$ in $\mathbb{R}^3$ as a linear combination of the vectors $u_1 = (1,1,1)$, $u_2 = (1,2,3)$ and $u_3 = (2, -1, 1)$. (5)

(c) Find the eigen values and eigen vectors for the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$. (5)

4. (a) Find the values of a, b and c such that

$\vec{F} = (x + 2y + az)\hat{i} + (bx - 3y - z)\hat{j} + (4x + cy + 2z)\hat{k}$

is irrotational. (5)

(b) Prove that $\left[\vec{a} \times \vec{b} \quad \vec{b} \times \vec{c} \quad \vec{c} \times \vec{a}\right] = \left[\vec{a}\vec{b}\vec{c}\right]^2$. (5)

(c) Show that $\nabla \times (\phi \vec{A}) = (\nabla \phi) \times \vec{A} + \phi(\nabla \times \vec{A})$. (5)

5. (a) If $\vec{F} = (x^2 + y^2)\hat{i} - 2xy\hat{j}$, evaluate $\int_C \vec{F} \cdot d\vec{r}$ where the curve C is the rectangle in xy plane bounded by $y = 0, x = a, y = b, x = 0$. (4)

(b) State Green's theorem. Verify Green's theorem in the plane for $\oint_C (3x^2 - 8y^2)\, dx + (4y - 6xy)\, dy$, where C is the boundary of the region defined by $y = \sqrt{x}$ and $y = x^2$. (2 + 5 = 7)

(c) If $\vec{F} = ax\hat{i} + by\hat{j} + cz\hat{k}$, where a, b, c are constants. Show that $\iint_S \vec{F} \cdot \hat{n}\, dS = \frac{4\pi}{3}(a + b + c)$, S being the surface of the sphere $(x-1)^2 + (y-2)^2 + (z-3)^2 = 1$. (4)

6. (a) Evaluate : (3 + 3 = 6)

(i) $L\left\{\frac{e^{at} - 1}{a}\right\}$

(ii) $L^{-1}\left\{\frac{1}{s-2} + \frac{2}{s+5} + \frac{6}{s^4}\right\}$.

(b) Evaluate $L\left\{\frac{\cos at - \cos bt}{t}\right\}$. (4)

(c) Apply convolution theorem to evaluate $L^{-1}\left\{\frac{s^2}{(s^2+a^2)}\right\}$. (5)

7. (a) Let $f(t)$ be a periodic function of period T so that $f(nT+t) = f(t)$ where $n = 0, 1, 2, 3, \ldots$. Prove that $L\{f(t)\} = \frac{1}{e^{-sT}}\int_0^T e^{-st} f(t)\,dt$. (5)

(b) Find the Laplace transform of the square wave function of period a defined by

$$f(t) = 1 \quad \text{when} \quad 0 < t < \frac{a}{2}$$

$$= -1 \quad \text{when} \quad \frac{a}{2} < t < a$$

(5)

(c) Solve $y'' + 4y' + 3y = e^{-t}$ if $y(0) = y'(0) = 1$ by Laplace transform. (5)
