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MA 171203

Roll No. of candidate | | | | | | | | | | |

2018

B.Tech. 2nd Semester End-Term Examination

ENGINEERING MATHEMATICS — II

(New Regulation)

Full Marks – 70 Time – Three hours

The figures in the margin indicate full marks for the questions.

Answer question No. 1 and any *four* from the rest.

1. In each of the following questions, four answers are provided of which only one is correct. Choose the correct answer. (10 × 1 = 10)

   (a) Diagonal elements of a skew-symmetric matrix are

   (i) 0 (ii) 1

   (iii) a non-zero constant (iv) any real number

   (b) The rank of a null matrix is

   (i) not defined (ii) –1

   (iii) 0 (iv) 1

[Turn over

(c) A square matrix A is idempotent if

(i) $A^2 = 0$ (ii) $A^2 = I$

(iii) $A^2 = A$ (iv) $A = A'$

(d) The sum of the eigen values of the matrix

$$A = \begin{pmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{pmatrix} \text{ is}$$

(i) 5 (ii) 6

(iii) 7 (iv) 12

(e) The dimension of the vector space $\mathbb{R}^3$ is

(i) 0 (ii) 1

(iii) 2 (iv) 3

(f) Let $\vec{a}$, $\vec{b}$ be two vectors. Then $[\vec{a}\ \vec{a}\, \vec{b}] =$

(i) ab (ii) a^2b

(iii) 1 (iv) 0

(g) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, then $div\, \vec{r} =$

(i) 3 (ii) 2

(iii) 1 (iv) 0

(h) Let $\vec{r} = (\sin t)\hat{i} + (\cos t)\hat{j} + t\hat{k}$ then $\left|\frac{d\vec{r}}{dt}\right| =$

(i) 0

(ii) 1

(iii) $\sqrt{2}$

(iv) $(\cos t)\hat{i} - (\sin t\, \hat{j}) + \hat{k}$

(i) $L\{e^{at}\}=$

(i) $\frac{1}{s-a}$ (ii) $\frac{1}{s+a}$

(iii) $\frac{s}{s-a}$ (iv) $\frac{s}{s+a}, s>a>0$

(j) $L^{-1}\left\{\frac{1}{s}\right\}=$

(i) 0 (ii) 1

(iii) 2 (iv) 3

2. (a) Let A, B be two symmetric matrices of the same size. Prove that AB is symmetric if and only if A, B commute. (2)

(b) Explain the matrix

$$A=\begin{pmatrix}1 & 5 & 7\\ 11 & 3 & 9\\ 5 & 7 & 13\end{pmatrix}$$

as the sum of a symmetric, and a skew-symmetric matrix. (3)

(c) Reduce the matrix

$$A=\begin{bmatrix}1 & 2 & 3 & 4\\ 2 & 1 & 4 & 3\\ 3 & 0 & 5 & -10\end{bmatrix}$$

to normal form, and find its rank. (5)

(d) Solve the following system of equations by Gauss elimination method (5)

$$x+y+z=3$$
$$x+2y+3z=4$$
$$x+4y+9z=6$$

3. (a) Find the eigen values, and the corresponding eigen vectors for the matrix

$$A = \begin{pmatrix} 1 & 3 \\ 4 & 5 \end{pmatrix}$$

Also write down the eigen values of A^2. (5)

(b) Verify Cayley-Hamilton theorem for the matrix

$A = \begin{pmatrix} 5 & 4 \\ 1 & 2 \end{pmatrix}$, and hence find A^{-1}. (5)

(c) Examine if the set

$$S = \{(0, 1, 1), (1, 0, 1), (1, 1, 0)\}$$

is a basis for $\mathbb{R}^3$. (5)

4. (a) Find the value of λ if the vectors

$$\vec{a} = \hat{i} + \hat{j} + \hat{k}$$
$$\vec{b} = 2\hat{i} + \hat{j} - \hat{k}$$
$$\vec{c} = \lambda\hat{i} - \hat{j} + \hat{k}$$

are coplanar. (3)

(b) Let $\vec{a}, \vec{b}, \vec{c}$ be three vectors. Show that

(i) $[\vec{a}+\vec{b} \quad \vec{b}+\vec{c} \quad \vec{c}+\vec{a}] = 2[\vec{a} \quad \vec{b} \quad \vec{c}]$

(ii) $\vec{a}\times(\vec{b}\times\vec{c})+\vec{b}\times(\vec{c}\times\vec{a})+\vec{c}\times(\vec{a}\times\vec{b})=\vec{0}$. (4 + 4 = 8)

(c) Find the shortest distance between the skew lines.

$\vec{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda\,(2\hat{i} + 3\hat{j} + 2\hat{k}),$ and

$\vec{r} = (2\hat{i} + 4\hat{j} + 5\hat{k}) + \mu\,(3\hat{i} + 4\hat{j} + 5\hat{k}),$ (4)

5. (a) If $\vec{r} = \vec{a}\sin wt + \vec{b}\cos wt$, where $\vec{a}, \vec{b}$ are constant vectors, and w is a scalar, prove that $\dfrac{d^2\vec{r}}{dt^2} = -w^2\vec{r}$. (2)

(b) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, $|\vec{r}| = r$, show that $grad\, r = \dfrac{1}{r}\vec{r}$. (3)

(c) Determine the constant λ so that the vector

$\vec{V} = (x+3y)\,\hat{i} + (y-2z)\,\hat{j} + (x+\lambda z)\,\hat{k}$ is solenoidal. (3)

(d) If $\vec{V} = xyz\hat{i} + 3x^2y\hat{j} + (xz^2 - y^2z)\hat{k}$, find $curl\,\vec{V}$ at the point $(1, -1, 1)$. (3)

(e) If $\vec{V} = 3xy\,\hat{i} - y^2\,\hat{j}$, evaluate $\int_c \vec{V}\cdot d\vec{r}$

between the points (0, 0) and (1, 2); where c is the parabola $y = 2x^2$. (4)

6. (a) Evaluate

(i) $L\{\cos^2 at\}$

(ii) $L\{t^3 e^{2t}\}$. (2 + 2 = 4)

(b) If $f(t) = e^{-2t}\sin 4t$,

Find $L\{f'(t)\}$. (3)

(c) Obtain $L^1\left\{\dfrac{3(s^2-1)^2}{2s^5}\right\}$. (3)

(d) State the convolution theorem. Using convolution theorem, find $L^{-1}\left\{\dfrac{1}{(s+a)(s+b)}\right\}$. (5)

7. (a) Find the Laplace transform of $f(t)$ defined by

$$f(t)=\begin{cases} t, & 0 \le t \le 3 \\ 0, & t > 3 \end{cases}.$$ (5)

(b) Let $f(t)$ be a periodic function of period T so that $f(nT+t)=f(t)$, where $n=0, 1, 2, 3, \ldots$

Find $L\{f(t)\}$. (5)

(c) Using Laplace transform, solve the differential equation

$$\frac{d^2x}{dt^2}-3\frac{dx}{dt}+2x=2e^{3t},$$

$x(0)=5, \quad x'(0)=7$. (5)
