Total No. of printed pages = 4

ECE 181401

Roll No. of candidate | | | | | | | | | |

2022

B.Tech. 4th Semester End-Term Examination

LINEAR ALGEBRA AND RANDOM PROCESSES

(New Regulation & New Syllabus)

Full Marks – 70 Time – Three hours

The figures in the margin indicate full marks for the questions.

Answer question No. 1 and any *four* from the rest.

1. Answer the following : $(10 \times 1 = 10)$

(i) A set of r n-vectors is said to be linearly independent if ——————

(ii) Norm of x is defined by $\|x\|$ and is given by ——————

(iii) λ is a characteristic root of a matrix A if and only if there exists a non zero vector X such that ——————

(iv) The image of a linear-mapping $\phi : V \rightarrow W$ is denoted by $\text{Im}(\phi)$ and is defined as ——————

(v) A real vector space V endowed with a real inner product defined on it is said to be —————— space.

(vi) Let A, B be any events with $P(AUB) = \frac{7}{8}$ $P(A \cap B) = \frac{1}{4}$ and $P(A^c) = \frac{5}{8}$. fIND $P(B)$.

(vii) Find the probability that at least one head appears in the throw of three fair coins.

(viii) Find the probability that there are 3 defective items in a sample of 100 items if 2% of items made in this factory are defective.

(ix) Define the cumulative distribution function $F(x)$ for a continuous random variable x.

(x) For a discrete random variable x, show that $\text{var}(kx) = k^2(\text{var}\, x)$.

2. (a) Let V be the vector space over the field F. Prove that the intersection of any collection of subspaces of V is a subspace of V. (3)

(b) Define a linear mapping. Prove that the mapping $\phi : R^3 \to R^2$ defined by $\phi(x, y, z) = (zx + y - z, x + y + z)$ is linear. (1 + 3 = 4)

(c) Define Basis and dimension of a vector space V. Prove that the set $S = \{(1,0,1),(0,1,1),(1,1,0)\}$ is a basis of R^3. (1 + 4 = 5)

(d) Let V be the set of all 2×2 matrices show that the set

$S = \left\{\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\right\}$ space V. (3)

3. (a) A box contains eight white and six red marbles. Find the probability of drawing two marbles of (i) same colour (ii) different colours. (2 + 2 = 4)

(b) In a test a student either guesses or copies or knows the answer to a multiple choice question with four choices. The prob that he guess is $\frac{1}{3}$ and copies it is $\frac{1}{4}$. Probability that his answer is correct provided he guess it is $\frac{1}{4}$ and that provided he copies it is $\frac{1}{8}$. Find the probability that he knew the answer given that he correctly answered it. (4)

(c) Determine the discrete probability distribution of a random variable X that denotes the minimum of the two numbers that appear when a pair of fair dice is thrown once. Also find the mean and variance. (2 + 1 + 1 = 4)

(d) Suppose a continuous random variable X has the following probability density function

$$f(x) = \begin{cases} k(1 - x^2) & for\ 0 < x < 1 \\ 0 & elsewhere \end{cases}$$

(i) Find k

(ii) Find $P(0.1 < x < 0.2)$

(iii) Find the mean of the distribution. (3)

4. (a) A linear mapping $\phi : R^3 \to R^2$ is defined by $\phi(x_1, x_2, x_3) = (3x_1 - 2x_2 + x_3,\ x_1 - 3x_2 - 2x_3)$, $(x_1, x_2, x_3) \in R^3$. Find the matrix of ϕ relative to the ordered basis $(0,1,1)$ $(1,0,1)$, $(1,1,0)$ or R^3 and $(1,0)$, $(0,1)$ of R^2. (3)

(b) Find all the eigen values and corresponding eigen vectors of the matrix

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}.$$ (6)

(c) Let V be the real inner product space. Find the value of $\langle 2u - 5v, 4u + 6v \rangle$ by using linearity property $\langle \alpha, \beta + \gamma \rangle = \langle \alpha, \beta \rangle + \langle \alpha, \gamma \rangle \forall \alpha, \beta, \gamma \in V$. (3)

(d) If α, β are any two vectors in a Euclidean space V, then prove that $\|\alpha + \beta\| \le \|\alpha\| + \|\beta\|$. (3)

5. (a) Find a non-zero vector w that is orthogonal to $u_1 = (1,2,1)$ and $u_2 = (2,5,4)$ in $\mathbb{R}^3$. (3)

(b) Show that the matrix $A = \begin{bmatrix} 0 & 2+3i \\ -2-3i & 0 \end{bmatrix}$ is skew-symmetric but is not skew-hermitian. (2)

(c) If a, b, c are all different and $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix} = 0$, show that $abc = -1$. (5)

(d) Solve by Cramer's rule :

$x + y + z = 0; 2x + 5y + 3z = 1; -x + 2y + z = 2$. (5)

6. (a) If A and B be any events with

$P(A) = \frac{1}{3}$, $P(B) = \frac{1}{4}$ and $P(A \cup b) = \frac{1}{2}$, find (i) $P(B / A^C)$ (ii) $P(A^C \cup B^C)$ (iii) $P(A^C \cap B^C)$. (3)

(b) Suppose three companies X, Y, Z produce TV's. X Produce twice as many as Y, while Y and Z produce the same number. It is known that 2% of X, 2% of Y and 4% of Z are defective. All the TV's produced are put together and then one TV is chosen at random

(i) What is the probability that the TV is defective?

(ii) Suppose a TV chosen is defective, what is the probability that this TV is produced by company X? (2 + 2 = 4)

(c) A man hits a target with probability $\frac{1}{4}$ (i) Determine the probability of hitting atleast twice when he fires 7 times (ii) How many times must be fire so that the probability of his hitting the target atleast once is greater than $\frac{2}{3}$? (2 + 2 = 4)

(d) In a factory producing blades, the probability of any blade being defective is 0.002. If blades are supplied in packets of 10, determine the number of packets containing (i) no defective (ii) two defective blades respectively in a consignment of 1000 packets. (2 + 2 = 4)

7. (a) The mean height of 500 male students in a certain college is 154 cm and the standard deviation is 15 cm. Assuming the height to be normally distributed find how many students have heights.

(i) between 120 cm and 150 cm

(ii) above 145 cm (3 + 2 = 5)

(b) If x is uniformly distributed in $-2 \leq x < 2$, find (i) $p(x<1)$ (ii) $P\left(|x-1| \geq \frac{1}{2}\right)$. (2 + 2 = 4)

(c) Let the mileage (in thousands of miles) of a particular tyre by a random variable x have the probability density function :

$$f(x)=\begin{cases}\frac{1}{20}e^{-x/20} & for\ x>0\\ 0 & for\ x\leq 0\end{cases}$$

Find the probability that one of these tyres will last

(i) at most 10000 miles

(ii) anywhere from 16000 to 24000 miles. (2 + 2 = 4)

(d) What do you mean by discrete random process? Give an example. (1 + 1 = 2)