Total No. of printed pages = 4

EC 131405

Roll No. of candidate

2019

B.Tech. 4th Semester End-Term Examination

RANDOM VARIABLES AND STOCHASTIC PROCESSES

(New Regulation) (w.e.f. 2017-18)

Full Marks – 70   Time – Three hours

The figures in the margin indicate full marks
for the questions.

Answer question No. 1 and any *five* from the rest.

1.   Answer *all* questions :   (10 × 1 = 10)

(a)   What is De Morgan's law?

(b)   What is duality principle?

(c)   What is Baye's Theorems?

(d)   Distinguish between continuous and discrete random variables.

(e)   Define n^{th} moment of random variable x.

(f)   Distinguish between a random variable and a random process

[Turn over

(g) Write the condition for two random variables to be uncorrelated

(h) Define autocorrelation function of a random process $x(t)$.

(i) What do you mean by a stationary process?

(j) What is an ergodic process?

2. (a) List at least four axioms of probability. (4)

(b) A universal set is given as $s = \{2, 4, 6, 8, 10, 12\}$. Define two subsets as $A = \{2, 4, 10\}$ and $B = \{4, 6, 8, 10\}$. Determine the following

   (i) $\overline{A} = S - A$,

   (ii) $A - B$ and $B - A$

   (iii) $A \cup B$

   (iv) $A \cap B$. (8)

3. (a) Define density function and distribution function of a random variable x. List the properties of density function. (6)

(b) Find the constant $b > 0$ such that the function $f_x(x) = \begin{cases} e^{3x/4}, & 0 \le x \le b \\ 0, & \text{otherwise} \end{cases}$ is a valid probability density. (6)

4. (a) A gaussian random variable x of zero mean and variance σ_x^2 is transformed by a square law device defined by $y = x^2$. Find the pdf of the new random variable y. (7)

(b) Plot the pdf and CDF of a gaussian distributed random variable and discuss about its density curve. (5)

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5. A random variable x is uniformly distributed with probability density function given by

$$f_x(x) = \begin{cases} k, & x_1 < x, < x_2 \\ 0, & \text{otherwise} \end{cases}$$

Find:

(a) the value of k

(b) CDF, $f(x)$ of x

(c) mean of x

(d) variance of x. $\hspace{2cm} (4 \times 3 = 12)$

6. (a) Show that variance of a random variable x is equal to $Var[x] = E[x^2] - E^2[x]$. $\hspace{1cm} (4)$

   (b) Define covariance of a random variable x. Let z be sum of two random variables x and y. Determine mean and variance of z. $\hspace{0.5cm} (2 + 6 = 8)$

7. (a) List the properties of Auto Correlation Function (acf). Show that acf has its maximum magnitude at the origin ($\tau = 0$). $\hspace{1cm} (6)$

   (b) Consider a random process $x(t)$ given by

   $$x(t) = A \cos(w_{ct} + \varphi)$$

   where w_c and φ are constants and A is a random variable. Determine whether $x(t)$ is Wide Sense Stationary (wss). $\hspace{1cm} (6)$

8. A random process is given by a randomly phased sinusoid. $x(t) = A \cos(w_c t + \Phi)$, where A and w_c are constants, while Φ is a random variable that is uniformly distributed over the range $(0, 2\pi)$.

   (a) Show that $x(t)$ is a stationary process in the wide sense

   (b) Show that $x(t)$ is ergodic in the mean

   (c) When the random variable Φ is replaced by a fixed φ_0, will the process by stationary?

   $(8 + 2 + 2 = 12)$

9. Write short notes on any *two* of the following :

   (a) Rayleigh distribution $(2 \times 6 = 12)$

   (b) Central limit theorem

   (c) Random signal

   (d) White noise.

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