Total No. of printed pages = 3

MA 171203

26/7/22

Roll No. of candidate | | | | | | | | | | |

2022

B.Tech. 2nd Semester End-Term Examination

ENGINEERING MATHEMATICS – II

(New Regulations)

Full Marks – 70 Time – Three hours

The figures in the margin indicate full marks for the questions.

Answer Question No. 1 and any *four* from the rest.

1. Choose the correct answer: $(10 \times 1 = 10)$

(i) For the matrices A and B, $(AB)^{-1} =$

(a) BA (b) $A^{-1}B^{-1}$

(c) $B^{-1}A^{-1}$ (d) none of these

(ii) The diagonal elements of a skew-symmetric matrix are

(a) 0 (b) 1

(c) a non-zero constant (d) any real number

(iii) The product of the eigen values of the matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ is

(a) 2 (b) 4

(c) 5 (d) 6

(iv) The dimension of the vector space $\mathbb{R}^3$ is

(a) 0 (b) 1

(c) 2 (d) 3

(v) $\nabla\phi$ of a scalar function ϕ is a vector __________ to the surface $\phi = c$

(a) parallel (b) tangent

(c) normal (d) none of these

(vi) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, then $div\, \vec{r} =$

(a) 0 (b) 1

(c) 2 (d) 3

(vii) If $\vec{r} = sin t\hat{i} + cos t\hat{j} + t\hat{k}$, then $\left|\frac{d\vec{r}}{dt}\right| =$ is

(a) 0 (b) 1

(c) $\sqrt{2}$ (d) 2

(viii) The vector $\vec{v}$ is called irrotational, if

(a) $div\, \vec{v} = 0$ (b) $curl\, \vec{v} = \vec{0}$

(c) $grad|\vec{v}| = 0$ (d) none of these

(ix) $L^{-1}\left\{\frac{1}{s+a}\right\} =$

(a) e^{-at} (b) e^{at}

(c) te^{-at} (d) te^{at}

(x) $f(x) = tan\, x$ is a periodic function with period

(a) $\frac{\pi}{2}$ (b) π

(c) 2π (d) $\frac{2\pi}{3}$

2. (a) Write the matrix $A = \begin{bmatrix} 4 & 5 & 1 \\ 3 & 7 & 2 \\ 1 & 6 & 8 \end{bmatrix}$ as $A = B + C$, where B is symmetric and C is skew-symmetric matrix. (5)

(b) Show that the matrix $A = \begin{bmatrix} cos\,\theta & sin\,\theta \\ -sin\,\theta & cos\,\theta \end{bmatrix}$ is orthogonal. (5)

(c) Find the inverse of $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$ by elementary row transformation. (5)

3. (a) Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 1 & -2 \\ -5 & 4 \end{bmatrix}$. (5)

(b) Examine if the set $S = \{(0,1,1),(1,0,1),(1,1,0)\}$ is a basis for $\mathbb{R}^3$. (5)

(c) Solve the following equation by Gauss-Jordan method: (5)

$x + y + z = 3$
$x + 2y + 3z = 4$
$x + 4y + 9z = 6$

4. (a) If $\vec{r} = \cos\omega t\hat{i} + \sin\omega t\hat{j}$, where ω is a scalar, prove that $\frac{d^2\vec{r}}{dt^2} = -\omega^2\vec{r}$. (3)

(b) Find the value of n, if the vector $r^n\vec{r}$ is solenoidal. (4)

(c) Find $div\vec{F}$ and $curl\vec{F}$ of the vector $\vec{F} = grad\left(x^3 + y^3 + z^3 - 3xyz\right)$. (4 + 4 =8)

5. (a) Find the directional derivative of $\phi(x,y,z) = xy^2 + yz^3$ at $(2,-1,1)$ in the direction of $\hat{i} - 2\hat{j} + 2\hat{k}$. (5)

(b) Evaluate $\int_c \vec{f}.d\vec{r}$ where $\vec{f} = y\hat{i} - x\hat{j}$, along the parabola $y = x^2$ from $(0,0)$ to $(1,1)$. (5)

(c) Use Green's Theorem to evaluate the integral $\oint_c \left(2x^2 - y^2\right)dx + \left(x^2 + y^2\right)dy$, where C in the boundary of the area enclosed by the x-axis and the semi circle $x^2 + y^2 = 1$ non the upper half of xy-plane. (5)

6. (a) Find the Laplace Transform of

(i) $f(t) = sin^2 t$

(ii) $f(t) = e^t\, cos t$ (2 + 3 =5)

(b) Evaluate $L^{-}\left\{\frac{1}{s-2} + \frac{2}{s+5} + \frac{3}{s^4}\right\}$. (3)

(c) Evaluate $\int_0^\infty \frac{e^{at} - e^{bt}}{t}dt$. (7)

7. (a) Apply convolution theorem to evaluate $L^{-}\left\{\frac{s^2}{\left(s^2+1\right)^2}\right\}$. (7)

(b) Solve the differential equation $\frac{d^2y}{dt^2} + 4y = 9t$, $y(0) = 0$, $y'(0) = 7$ using Laplace transform. (8)