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ECE 181401

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Roll No. of candidate

2023

B.Tech. 4th Semester End-Term Examination

LINEAR ALGEBRA AND RANDOM PROCESS

New Regulation (w.e.f. 2017-18) & New Syllabus (w.e.f. 2018-19)

Full Marks – 70 Time – Three hours

The figures in the margin indicate full marks for the questions.

Answer Question No. 1 and any *four* from the rest.

1. Choose the correct option : $(10 \times 1 = 10)$

(i) Let for a square matrix A, $|A| = 0$. Then which of the following is wrong?

(a) A is singular (b) A can have a zero row

(c) A is invertible (d) A can have a zero column

(ii) Let S and T be two subspaces of a vector space V over the field F. Then $S \cup T$ will always a subspace of $V(F)$ if

(a) $S \cap T = \phi$

(b) S and T contained in each other

(c) S has the additive identity of $V(F)$

(d) None of these

(iii) Which of the following symmetric matrix is positive definite?

(a) $\begin{bmatrix} 2 & 3 \\ 3 & 2 \end{bmatrix}$ (b) $\begin{bmatrix} 8 & -2 \\ -2 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & -3 \\ -3 & -5 \end{bmatrix}$

[Turn over

(iv) Which of the following is a linear map on $\mathbb{R}^2$?

(a) $F(x, y)=(xy, x)$ (b) $F(x, y)=(0, y+3)$

(c) $F(x, y)=(x^2, y)$ (d) $F(x,y)=(x+y, x)$

(v) Let V be a vector space of finite dimension n. Then any $n+1$ or more vectors must be linearly dependent

State True/False

(vi) Let a coin is tossed twice. Then the probability of getting at most one head is

(a) $\frac{1}{4}$ (b) $\frac{3}{4}$

(c) $\frac{1}{2}$ (d) None of these

(vii) If A and B are independent events. Then

(a) $P(A \cap B)=0$ (b) $P(A \cap B)=P(A).P(B)$

(c) $P(A \cap B)=P(A)+P(B)$ (d) None of these

(viii) If $T: V \to W$ be any linear transform then, Kernel of T is

(a) a subspace of V (b) a subspace of W

(c) not a subspace (d) none of these

(ix) The standard deviation of binomial distribution is

(a) np (b) npq

(c) $\sqrt{npq}$ (d) none of these

(x) If the probability density function of continuous random variable X is given by $f_x(x)=1;\ 0<x<1$. Then the value of $E(x)$ is

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) $\frac{2}{5}$ (d) none of these

2. (a) Define a vector space. Give two examples. (3+2=5)

(b) Define subspace of a vector space. Consider the vector space $\mathbb{R}^3$ ($\mathbb{R}$). Let $W=\{(a, b, c) \in \mathbb{R}^3 | a=2b=3c\}$ be a subset of $\mathbb{R}^3$. Check whether W is a subspace of $\mathbb{R}^3$ (R). (3+3=6)

(c) Determine whether or not the following vectors in $\mathbb{R}^3$ are linearly independent. $(1, -2, 1), (2, 1, -1), (7, -4, 1)$. (4)

3. (a) State Rank – Nullity theorem. Verify Rank – Nullity theorem with the linear transformation

T: $\mathbb{R}^2 \to \mathbb{R}^2$ defined by $T(x, y) = (x, x+y, y)$. (2+3=5)

(b) Suppose u, v, w are linearly independent vectors. Prove that S is linearly independent where $S = \{u+v-2w, u-v-w, u+w\}$. (5)

(c) Verify Cayley – Hamilton theorem for the matrix $A = \begin{bmatrix} 5 & 3 \\ 2 & 10 \end{bmatrix}$. (5)

4. (a) Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$.

(2+3=5)

(b) Define an inner product space. Give an example. (2+1=3)

(c) Find M^3 for $M = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 2 & 3 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 4 & 5 \end{bmatrix}$ using block matrix multiplication. (5)

(d) Define orthogonal vectors in an inner product space. Find the value of k so that the vectors $u = (1, 2, k, 3)$ and $v = (3, k, 7, -5)$ in $\mathbb{R}^4$ are orthogonal. (2)

5. (a) In a class 60% of the students are boys and remaining are girls. It is known that the probability of a boy getting distinction is 0.30 and that of girl getting distinction is 0.35. Find the probability that a student chosen at random will get distinction. (5)

(b) The joint probability mass function of random variables X and Y is given by $P_{x,y}(x_i, y_j) = \begin{cases} k\, x_i\, {y_j}^2, & x_i = 1, 2, 3;\ y_j = 1, 2\ 3 \\ 0, & otherwise \end{cases}$

Find the marginal density mass functions of X and Y. (5)

(c) A random variable X has the density function $f(x) = \frac{c}{x^2+1}$ where $-\infty < x < \infty$. Find

(i) the value of C

(ii) $P\left(\frac{1}{3} < x < 1\right)$. (5)

6. (a) Find the mean and variance of Binomial distribution. (3+3=6)

(b) In a normal distribution 31% of the items are under 45 and 8% of the items are over 64. Find the mean and standard deviation of the distribution. (5)

(c) Let $T : U \to V$ be a linear mapping show that Kernel of T, ker T is a subspace of U where U are V are vector spaces over the same field F. (4)

7. (a) What do you mean by Random Processes? Write briefly about the types of random processes. (2+4=6)

(b) In the fair – coin experiment, a random process $X(t)$ is defined as follows :

$$x(t) = \cos \pi t, \quad \text{if heads occur}$$
$$= t, \quad \text{if tails occur}$$

Find (i) $E[x(t)]$, (ii) $F_x(x, t)$ for $t = 0.25, 0.5, 1$. (4)

(c) A dice is tossed and corresponding to the dots $S = \{1, 2, 3, 4, 5, 6\}$. A random process $X(t)$ is formed with the following time functions : (5)

$X(1, t) = -3,\ X(2, t) = 1,\ X(3, t) = 1 - t$

$X(4, t) = 1 + t,\ X(5, t) = 2 - t,\ X(6, t) = t - 2$

Find $\mu_x(t),\ E(X^2),\ \sigma^2 x(t),\ R_{XX}(t_1, t_2)$.