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Roll No. of candidate ☐☐☐☐☐☐☐☐☐☐

2023

B.C.A. 2nd Semester End-Term Examination

MATHEMATICS – II

Full Marks – 70   Time – Three hours

The figures in the margin indicate full marks for the questions.

Answer question No. 1 and any *four* from the rest.

1. Choose the correct answer : $(10 \times 1 = 10)$

    (i)   Which of the following is not a null set?

    (a) $\{x : x \neq x\}$

    (b) $\{\phi\}$

    (c) ϕ

    (d)   the set of natural number less than 1

    (ii)   Let A be a finite set having m elements, and B be a finite set having n elements. The number of relations from A to B is

    (a) mn (b) $m + n$

    (c) 2^{m+n} (d) 2^{mn}

    (iii) If $A \subseteq B$, then

    (a) $A \cup B = A$ (b) $A \cup B = B$

    (c) $A \cup B \subseteq A$ (d)   None of these

    (iv) $^{10}P_9 =$

    (a)   1   (b)   10

    (c)   10!   (d) $\dfrac{10!}{9!}$

[Turn over

(v) $^nC_r =$

(a) $\dfrac{n!}{(n-r)!}$

(b) $\dfrac{n!}{r!\,(n-r)!}$

(c) $\dfrac{(n-r)!}{r!}$

(d) $\dfrac{n!\,r!}{(n-r)!}$

(vi) The common difference of the series with terms $1, -3, -7, -11,....$ is

(a) -2

(b) 2

(c) -4

(d) 4

(vii) If the series $u_1 + u_2 + u_3 + ... + u_n + ...$ is convergent, then

(a) $\underset{n \to \infty}{Lt}\ u_n = 0$

(b) $\underset{n \to \infty}{Lt}\ u_n = 1$

(c) $\underset{n \to \infty}{Lt}\ u_n < 1$

(d) $\underset{n \to \infty}{Lt}\ u_n > 1$

(viii) A vertex is called a pendant vertex if the degree of the vertex is

(a) 0

(b) 1

(c) 2

(d) 3

(ix) A complete graph with 5 vertices has —————— edges.

(a) 5

(b) 7

(c) 10

(d) 12

(x) Which of the following is not a singleton set?

(a) $\{\phi\}$

(b) $\{x : x + 10 = 10\}$

(c) $\{x : x^2 = 9\}$

(d) the set of even but prime numbers

2. (a) Let A, B be two sets. Prove that (8)

(i) $(A \cup B)^C = A^C \cap B^C$

(ii) $(A \cap B)^C = A^C \cup B^C$

(b) If $A \subseteq B$, $B \subseteq C$ prove that $A \subseteq C$. (3)

(c) Prove with the help of mathematical induction, that (4)

$1 + 3 + 5 + ... + (2n - 1) = n^2$

3. (a) Prove that : (6)

(i) $A \cap \varphi = \varphi$ (ii) $A \cup U = U$

U being the universal set.

(b) Let $A = \{3, 4, 5\}$ and

$$R = \{(3, 4), (4, 3), (5, 4), (5, 3)\}$$

Is the relation R transitive? Explain. (3)

(c) Prove that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$. (6)

4. (a) Determine the value of (4)

(i) $^9P_3$ (ii) $^{10}C_6$

(b) Determine the value of n if (5)

$3 \times {}^nP_4 = 7 \times {}^{n-1}P_4$.

(c) How many 6 digit numbers can be formed from the digits 0, 1, 2, 3, 4, 5, 6, 7 if no digit is repeated? (6)

5. (a) A committee of 5 people is to be chosen from a group of 6 men and 4 women. How many committees are possible if there are to be 3 men and 2 women? (6)

(b) Show that (6)

$${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r.$$

Where $n \geq r \geq 1$ and n and r are natural numbers.

(c) How many arrangement of the letters of the word REMAND are possible if there are no restriction? (3)

6. (a) Give an example of a group containing a loop and parallel edges. (2)

(b) State and prove the Handshaking theorem. (6)

(c) Prove, in a graph that the number of vertices of odd degree is even. (7)

7. (a) A graph contains 35 edges, 4 vertices are of degree 5 each, 5 vertices are of degree 4 each. 4 vertices are of degree 3 each, and the remaining vertices are of degree 2 each. Find the total number of vertices. Also find the number of vertices with degree 2. (6)

(b) Let G be a simple graph with n vertices. Show that a degree of a vertex of G cannot exceed $(n-1)$. (4)

(c) Give an example of a graph which is Hamiltonian as well as Eulerian. (5)