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MA 181301 A

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2023

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B.Tech. 3rd Semester End-Term Examination

CE, ME, PEIE

MATHEMATICS III-A

(New Regulation and New Syllabus)

Full Marks – 70

Time – Three hours

The figures in the margin indicate full marks for the questions.

Answer Question No. 1 and any *four* from the rest.

1. Answer the following (Fill in the blanks) : (10 × 1 = 10)

(i) The differential equation of $z = f\left(\frac{y}{x}\right)$ is _____.

(ii) Solution of $\frac{\partial^2 z}{\partial x \partial y} = \frac{1}{xy}$ is _____.

(iii) The complete solution of the differential equation $z = px + qy + \sqrt{1 + p^2 + q^2}$ is _____.

(iv) The one dimensional heat equation is _____.

(v) If $L\{F(t)\} = f(p)$, then $L\{F(at)\} =$ _____.

(vi) If $L^{-1}\{f(p)\} = F(t)$ then $L^{-1}\{f(ap)\} =$ _____.

[Turn over

(vii) Given that $P(A) = \frac{3}{8}$, $P(B) = \frac{5}{8}$ and $P(A \cup B) = \frac{3}{4}$, then $P\left(\frac{A}{B}\right) = \text{_____}$.

(viii) If a random variable X can take the values 0, 1 and 2 with probability $\frac{1}{4}$, $\frac{1}{2}$ and $\frac{1}{4}$ respectively then the expected value of X i.e. $E(X) = \text{_____}$.

(ix) In a binomial distribution, if n be the number of trials and p be the probability of a success and q that of a failure in a single trial, then the standard deviation of the binomial distribution is _____.

(x) In χ^2 test, the observed and theoretical frequencies agree exactly if $\chi^2 = \text{_____}$.

2. Answer the following :

(a) Derive a partial differential equation by eliminating the arbitrary function from $f(x+y+z, x^2+y^2+z^2) = 0$. (3)

(b) Solve the following : (3 × 4 = 12)

(i) $x^2(y-z)p + y^2(z-x)q = z^2(x-y)$

(ii) $4xyz = pq + 2px^2y + 2qxy^2$.

(iii) $z^2(p^2x^2 + q^2) = 1$.

3. Answer the following :

(a) Use the method of separation of variables to solve the equation

$$\frac{\partial^2 u}{\partial x^2} - 2 \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0. \quad (5)$$

(b) Four cards are drawn from a pack of 52 cards. Find the probability that two are spades and two are hearts. (3)

(c) Assume that the probability of an individual coalminer being killed in a mine accident during a year is $\frac{1}{2400}$. Use Poisson's distribution to calculate the probability that in a mine employing 200 miners there will be at least one fatal accident in a year. (3)

(d) A random sample of 10 boys had the I. Q's 70, 120, 110, 101, 88, 83, 95, 107 and 100. Do these data support the assumption of a population mean I.Q. of 160? (4)

4. Answer the following :

- (a) In a bolt factory there are four machines A, B, C, D manufacturing 20%, 15%, 25%, 40% of the total Output respectively. Of their outputs 5%, 4%, 3% and 2% in the same order are defective bolts. A bolt is chosen at random from the factory's production and is found to be defective. What is the probability that the bolt was manufactured by machine A or machine D? (6)
- (b) In a normal distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and standard deviation of the distribution. (2 + 3 = 5)
- (c) A bag contains defective articles, the exact number of which is not known. A sample of 100 from the bag gives 10 defective articles. Find the limits for the proportion of defective articles in the bag. (4)

5. Answer the following :

- (a) Apply Charpit's method to solve $2xz - px^2 - 2qxy + pq = 0$. (6)
- (b) Find $L \{e^{-t} \cos t \cos 2t\}$. (3)
- (c) Find $L^{-1} \left\{ \frac{2p+1}{p^2-4} \right\}$. (3)
- (d) Discuss the joint probability distribution. (3)

6. Answer the following :

- (a) Find $L^{-1} \left\{ \frac{5p+3}{(p-1)(p^2+2p+5)} \right\}$. (4)
- (b) Solve by using Laplace transform $\frac{d^2x}{dt^2} + 9x = \cos 2t$ if $x(0) = 1$, $x\left(\frac{\pi}{2}\right) = -1$. (7)
- (c) Fit a straight line to the following data : (4)

$$x: 10 \ 20 \ 30 \ 40 \ 50$$

$$y: 7 \ 12 \ 18 \ 22 \ 28$$

7. Answer the following :

(a) Solve the wave equation $\frac{\partial^2 y}{\partial x^2} = \frac{1}{c^2} \times \frac{\partial^2 y}{\partial t^2}$ the boundary and initial conditions are $y(0, t) = y(l, t) = 0$, $y(x, 0) = f(x)$ and $\frac{\partial y}{\partial t} = 0$ when $t = 0$. (7)

(b) Define Markov process. Draw graphs for the Markov chain with the following transition probability matrices. (1 + 3 = 4)

$$\begin{matrix} & a_1 & a_2 & a_3 \\ a_1 & \left(\begin{array}{ccc} 1/2 & 1/2 & 0 \\ 0 & 1/2 & 1/2 \\ 1/2 & 1/4 & 1/4 \end{array} \right) \\ a_2 & & & \\ a_3 & & & \end{matrix}$$

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(c) Use convolution theorem to evaluate $L^{-1} \left\{ \frac{p}{(p^2 + 4)^2} \right\}$. (4)