Total No. of printed pages = 4

MA 181202

28/6/23

Roll No. of candidate | | | | | | | | | | |

2023

B.Tech. 2nd Semester End-Term Examination

MATHEMATICS – II

New Regulation (w.e.f. 2017-18) & New Syllabus (w.e.f. 2018-19)

Full Marks – 70 Time – Three hours

The figures in the margin indicate full marks for the questions.

Answer question No. 1 and any *four* from the rest.

1. Select the correct answer : $(10 \times 1 = 10)$

(i) If $f(x, y, z) = xyz$, then $grad\, f$ at the point $(1, 1, 1)$ is

(a) $\hat{i} + \hat{j} + \hat{k}$ (b) $2\hat{i} + \hat{j} + \hat{k}$

(c) $\hat{i} + 2\hat{j} + 2\hat{k}$ (d) $\hat{i} + \hat{j} + 2\hat{k}$

(ii) If $\vec{a}$ and $\vec{b}$ are irrotational then $\vec{a} \times \vec{b}$ is

(a) irrotational (b) solenoidal

(c) unit vector (d) none of these

(iii) The general solution of the differential equation $\dfrac{d^2y}{dx^2} - 4y = \sin 2x$ is

(a) $y = (c_1 + c_2x)e^{2x} - \dfrac{1}{8}\sin 2x$

(b) $y = (c_1 + c_2x)e^{2x} + \dfrac{x}{8}\sin 2x$

(c) $y = c_1 e^{2x} + c_2 e^{-2x} - \dfrac{1}{8}\sin 2x$

(d) $y = c_1 e^{2x} + c_2 e^{-2x} + \dfrac{1}{8}\sin 2x$

[Turn over

(iv) The integrating factor for the differential equation $\frac{dy}{dx}+\frac{x}{2(1-x^2)}y=\frac{x}{2}$ is

(a) $(1-x^2)^{-\frac{1}{4}}$ (b) $\sqrt{(1-x^2)}$

(c) $\log(1-x^2)$ (d) None of these

(v) $\frac{d}{dx}\{xJ_1\}$ is equal to

(a) $xJ_0(x)$ (b) $J_1(x)$

(c) $-x^{-1}J_2(x)$ (d) None of these

(vi) The value of $xJ_{n1}(x) =$

(a) $-nJ_n(x)-xJ_{n-1}(x)$ (b) $-nJ_n(x)+xJ_{n-1}(x)$

(c) $-nJ_n(x)-xJ_{n+1}(x)$ (d) $-nJ_n(x)+xJ_{n+1}(x)$

(vii) The transformation $w=\frac{1}{z}$ maps the circle $|z|=1$ to a

(a) circle (b) plane

(c) ellipse (d) none

(viii) If v is the harmonic conjugate of u then

(a) $u_y=v_x$ (b) $u_x=v_y$

(c) $u_x=-v_y$ (d) $u_y=-v_x$

(ix) If $f(z)$ is an integral function satisfying the inequality $|f(z)|\leq M$ where M is a real number then $f(z)$ is

(a) a function containing both z and $\overline{z}$

(b) a function containing only z

(c) a constant function

(d) none of these

(x) The function $f(z)=\frac{\sin z}{z}$ has

(a) a removable singular point at $z=0$

(b) an essential isolated singular point at $z=0$

(c) both removable and essential isolated singular point at $z=0$

(d) none of these

2. (a) Find the unit tangent vector to the curve $x = t$, $y = t^2$, $z = t^3$ at the point $t = 1$. (3)

(b) Find the directional derivative of the function $f = x^2 - y^2 + 2z^2$ at the point $P(1, 2, 3)$ in the direction of the line PQ where Q is the point (5,0,4). In what direction it will be maximum? Find also the magnitude of this maximum. $(3+\frac{1}{2}+\frac{1}{2}=4)$

(c) Find the work done is moving a particle in the force field $\vec{F} = 3xy\hat{i} - y^2\hat{j}$ along the parabola $y = 2x^2$ from (0, 0) to (1, 2). (4)

(d) (i) Find the constants a, b and c so that $\vec{F} = (x + 2y + az)\hat{i} + (bx - 3y - z)\hat{j} + (4x + cy + 2z)\hat{k}$ is irrotational. (4)

Or

(ii) Show that $\nabla^2 f(r) = f''(r) + \frac{2}{r} f'(r)$.

3. (a) Find the differential equation of all circles of radius r whose centre's lie on the x - axis. (3)

(b) Solve (any *three*) : $(3 \times 4 = 12)$

(i) $(1 + y^2)\,dx = (\tan^{-1} y - x)\,dy$

(ii) $x\,dy + y\,dx + \dfrac{x dy - y dx}{x^2 + y^2} = 0$

(iii) $e^y (1 + x^2)\dfrac{dy}{dx} - 2x(1 + e^y) = 0$

(iv) $x^2 p^2 + xyp - 6y^2 = 0$

(v) $y = x + 2\tan^{-1} p$.

4. (a) Solve the differential equation (4)

$(D^2 - 4D + 3)\,y = e^x \sin 2x$

(b) Solve the series the equation (7)

$(1 + x^2)\dfrac{d^2 y}{dx^2} + x\dfrac{dy}{dx} - y = 0$

(c) (i) Show that

$$J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \sin x.$$

Or

(ii) Express $f(x) = x^4 + 3x^3 - x^2 + 5x - 2$ in terms of Legendre polynomials. (4)

5. (a) Evaluate $\oint_S \vec{F}.\hat{n}\, ds$ where $\vec{F} = yz\hat{i} + zx\hat{j} + xy\hat{k}$ and S is the part of the surface $x^2 + y^2 + z^2 = 1$ which lies in the 1st octant. (5)

(b) Use Stoke's theorem to evaluate $\int_S \left(\nabla \times \vec{F}\right) \hat{n}\, ds$ where $\vec{F} = y\hat{i} + (x - 2xz)\hat{j} - xy\hat{k}$ and S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$ above the xy - plane. (5)

(c) Evaluate using calculus of residues $\int_0^{2\pi} \frac{d\theta}{5 - 3\cos\theta}$. (5)

6. (a) Show that $u(x, y) = x^3 - 3xy^2$ is a harmonic function. Find the harmonic conjugate of u and the analytic function $f(z) = u + iv$. (1+3+1=5)

(b) Show that the function $f(z) = \sqrt{|xy|}$ is not analytic at the origin, although the Cauchy- Riemann equations are satisfied at the origin. (4)

(c) Find the region in the $w-$plane into which the rectangular region in the $z-$ plane bounded by $x = 0$, $y = 0$, $x = 1$ and $y = z$ is mapped under the transformation $w = z + (2 - i)$. (3)

(d) Expand $f(z) = \log(1 + z)$ is powers of z. Also state the region of validity of the series. (2+1=3)

7. (a) Evaluate $\int_{1-i}^{2+i} (2x + iy + 1)dz$ along (3+3=6)

(i) the straight line joining $(1 - i)$ to $(2 + i)$

(ii) the curve $x = t + 1$, $y = 2t^2 - 1$.

(b) Evaluate : (2+3=5)

(i) $\oint_C \frac{z^2 + 5}{z - 3} dz$ where c is the circle $|z| = 4$.

(ii) $\oint_C \frac{e^{2x}}{(z + 1)^4} dz$, where C is the circle $|z| = 2$.

(c) Use Residue theorem to evaluate $\oint_C \frac{2z - 1}{z(z + 1)(z - 3)} dz$ where C is the circle $|z| = 2$. (4)
