Total No. of printed pages = 8

MA 181202

Roll No. of candidate

2019

B.Tech. 2nd Semester End-Term Examination

MATHEMATICS – II

(New Regulation)

(w.e.f. 2017–18 and New Syllabus)

Group A – (w.e.f. 2018–2019)

Full Marks – 70 Time – Three hours

The figures in the margin indicate full marks for the questions.

Answer question No. 1 and any *four* from the rest.

1. Choose the correct answer from the following : (10 × 1 = 10)

(i) The tangent vector to the curve $x = t^2 - 1$, $y = 4t - 3$, $z = 2t^2 - 6t$ at $t = 1$ is given by

(a) $2\hat{i} + 4\hat{j} - 2\hat{k}$

(b) $2\hat{i} - 4\hat{j} - 2\hat{k}$

(c) $\hat{i} + \hat{j} - \hat{k}$

(d) none of these

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(ii) The maximum value of the directional derivative of $\phi = x^2 - 2y^2 + 4z^2$ at the point $(1, 1, -1)$ is

(a) $\frac{3\sqrt{3}}{7}$ (b) $6\sqrt{\frac{7}{3}}$

(c) $\sqrt{84}$ (d) None of these

(iii) The value of λ so that the vector $\vec{v} = (x+3y)\hat{i} + (y-2z)\hat{j} + (x+\lambda z)\hat{k}$ is a solenoidal vector is

(a) -2 (b) 1

(c) 3 (d) none of these

(iv) The differential equation

$(ay^2 + x + x^8)dx + (y^8 - y + bxy)dy = 0$ is exact, if

(a) $a = 1,\ b = 3$

(b) $a = b$

(c) $b = 2a$

(d) $2b = a$

(v) The solution of the clairautis equation

$$y + e^p = px \text{ is}$$

(a) $y = cx + e^c$

(b) $y = cx - e^c$

(c) $x = cy - e^c$

(d) $x = cy + e^{-c}$

(vi) The integrating factor of the differential equation $(x^3 + y^3)dx = xy^2dy$ is

(a) $\frac{1}{xyz}$ (b) $\frac{1}{y^2}$

(c) $\frac{1}{y^4}$ (d) $\frac{1}{x^4}$

(vii) The Rodrigue's formula for $P_n(x)$, The legendre's polynomial of degree n is $P_n(x) = K\frac{d^n}{dx^n}(x^2-1)^n$ where

(a) $K = \frac{n!}{2^n}$ (b) $K = \frac{2^n}{n!}$

(c) $K = \frac{1}{2^n n!}$ (d) $K = \frac{1}{2^n (n!)^2}$

(viii) The series

$x - \frac{x^3}{2^2(1!)^2} + \frac{x^5}{2^4(2!)^2} - \frac{x^7}{2^6(3!)^2} + \ldots\infty$ equal

(a) $J_0(x)$ (b) $J_{1/2}(x)$

(c) $xJ_0(x)$ (d) $xJ_{1/2}(x)$

(ix) The value of the integral $\int_C \frac{z^2 - z + 1}{z - 1} dz$, where C is the circle $|z| = \frac{1}{2}$, is

(a) 0 (b) πi

(c) $-\pi i$ (d) none of these

(x) The bound of the integral $\left|\int_C \frac{dz}{z}\right|$, where C is C is the circle $|z| = r$, is

(a) π (b) 2π

(c) $2\pi r$ (d) $\log(r)$

2. (a) Show that $\nabla r^n = nr^{n-2}\vec{r}$ where $\overline{r} = x\hat{i} + y\hat{j} + z\hat{k}$. (2)

(b) Show that $div(curl\ \vec{v}) = 0$. (3)

(c) Show that the vector field represented by $\overline{F} = (z^2 + 2x + 3y)\hat{i} + (3x + 2y + z)\hat{j} + (y + 2zx)\hat{k}$ is irrotational. Obtain a scalar function ϕ such that $\vec{F} = grad\ \phi$. (5)

(d) (i) Evaluate $\iint_S \vec{F} \cdot \hat{n}\ ds$ over the entire surface of the region above the xy–plane bounded by the cone $z^2 = x^2 + y^2$ and the plane $z = 4$, if $\vec{F} = 4xz\hat{i} + xyz^2\hat{j} + 3z\hat{k}$. (5)

Or

(ii) Apply Stoke's theorem to evaluate $\int_C \vec{F} \cdot d\overline{r}$ where $\overline{F} = y^2\hat{i} + xy\hat{j} + xz\hat{k}$ and C is bounding curve of the hemisphere $x^2 + y^2 + z^2 = 9,\ z > 0$ oriented in the positive direction. (5)

3. (a) Solve the differential equation

$$\frac{dy}{dx} - y\tan x = 3e^{-\sin x}. \quad (3)$$

(b) Solve any *three* of the following differential equations : (4 × 3)

(i) $\frac{dy}{dx} + x\sin 2y = x^3 \cos^2 y$

(ii) $y(2xy+1)dx + x(1+2xy-x^3y^3)dy = 0$

(iii) $y = 2px + y^2p^3$

(iv) $x^2p^2 + 3xyp + 2y^2 = 0$, where $p = \frac{dy}{dx}$.

4. (a) Solve any *one* of the following differential equations : (4)

(i) $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = e^{2x}$

(ii) $\frac{d^2y}{dx^2} - 4y = \sin 2x$.

(b) Solve in series the equation

$$2x^2\frac{d^2y}{dx^2} + (2x^2 - x)\frac{dy}{dx} + y = 0. \quad (7)$$

(c) Prove that (any *one*) (4)

(i) $J_{-n}(x) = (-1)^n J_n(x)$

(ii) $P_n(-x) = (-1)^n P_n(x)$.

5. (a) State and prove the necessary conditions for an analytic function $f(z) = u(x, y) + iv(x, y)$ in a domain D of the complex plane. (5)

(b) (i) If $f(z) = \dfrac{xy^2(x+iy)}{x^2+y^4}$, $z \neq 0$

$= 0$, $\quad z = 0$.

Show that the Cauchy-Riemann equations are satisfied at the origin. Yet $f'(0)$ does not exist uniquely. (5)

Or

(ii) Show that $u(x, y) = e^x(\cos y - \sin y)$ is harmonic. Determine its harmonic conjugate $v(x, y)$ and hence the analytic function $f(z) = u + iv$. (2 + 3)

(c) What is Mobius transformation? Show that the transformation $w = i\dfrac{1-z}{1+z}$ transforms the circle $|z| = 1$ into the real axis of the w–plane and the interior of the circle, $|z| < 1$ into the upper half of the w–plane. (1 + 4)

6. (a) By using Cauchy's integral formula for derivatives, evaluate $\oint_C \frac{e^{-2z}}{(z+1)^3}\,dz$, where C is the circle $|z| = 2$. (3)

(b) (i) Expand $f(z) = \frac{1}{(z+1)(z+3)}$ in a Taylor's/Laurent's series valid for the regions : (4)

(1) $|z| < 1$

(2) $1 < |z| < 3$

Or

(ii) State the Taylor's theorem. Expand $f(z) = \sin z$ in a Taylor's series in powers of $(z - \pi/4)$. (1 + 3)

(c) By using Cauchy's Residue theorem, evaluate $\oint_C \frac{2z-1}{z(z+1)(z-3)}\,dz$, where C is the circle $|z| = 2$.

(3)

(d) Apply the method of contour integration to evaluate (any one) : (5)

(i) $\int_0^{2\pi} \frac{d\theta}{1 - 2a\cos\theta + a^2}$, $0 < a < 1$

(ii) $\int_{-\infty}^{\infty} \frac{x^2 dx}{(x^2+1)(x^2+4)}$.

7. (a) Use Green's theorem in a plane to evaluate the integral $\oint_C \left[2x^2 - y^2 dx + \left(x^2 + y^2\right)dy\right]$, where C is the boundary in the xy–plane of the area enclosed by the x–axis and the semi-circle $x^2 + y^2 = 1$ in the upper half xy–plane. (5)

(b) Solve the differential equation

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = xe^{3x}.$$ (4)

(c) Express $f(x) = x^3 - 5x^2 + x + 2$ interms of Legendre's polynomial.

(d) Prove that (any one) : (3)

(i) $\frac{d}{dx}\left[x^{-n} J_n(x)\right] = -x^{-n} J_{n+1}(x)$.

(ii) $xP_n' - P_{n-1}' = nP_n$.